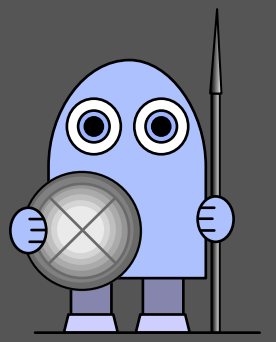
no problem !

book 2

problem book

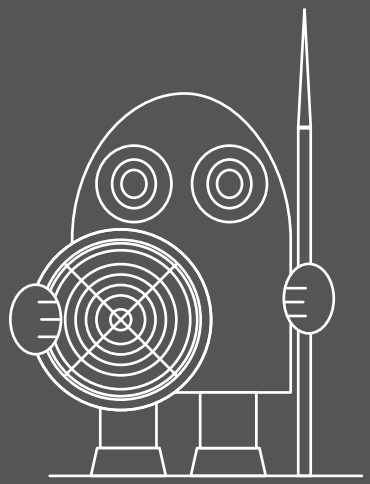
four winds



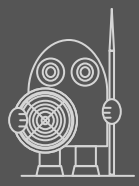
four winds maths

First published 2021

www.fourwindsmaths.com



list of problems



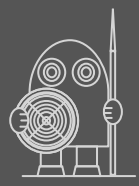
PTO

KEY

problems on logic, sets, combinations, permutations, probability, statistics

miscellaneous number problems (based on pre-algebra skills)

problems involving various aspects of shape (2-D and 3-D)

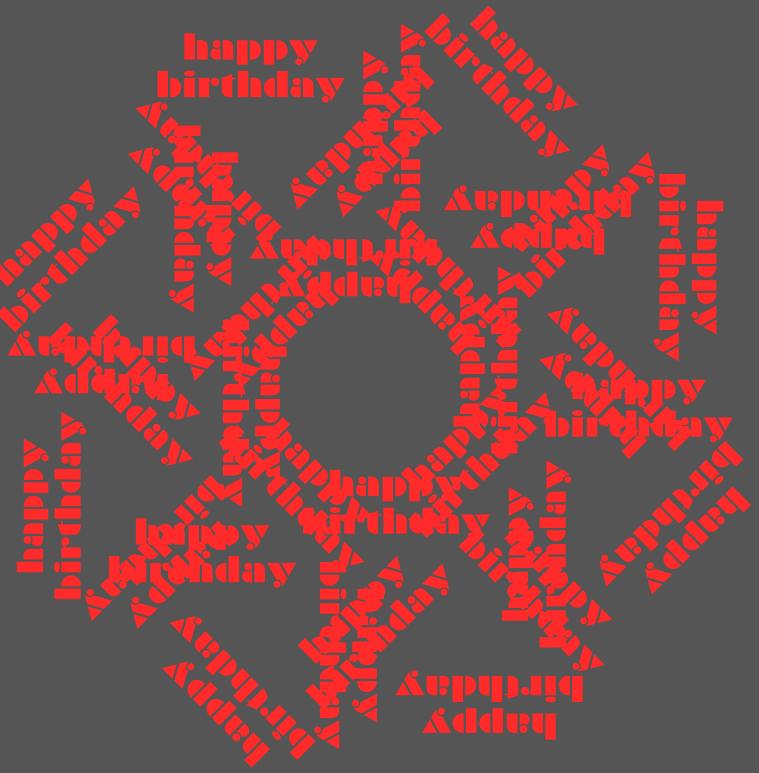


mark time . . .

Susan and her younger brother Mark both have their birthdays today. Here are two facts about their ages :

- Today Susan is three times as old as Mark.
- In two years' time, Susan's age will be exactly double Mark's age.

From these two facts you can work out everything you need to know about the ages of the brother and sister. Your problem is this : how old will Susan and Mark be in two years' time?



2 shopping day

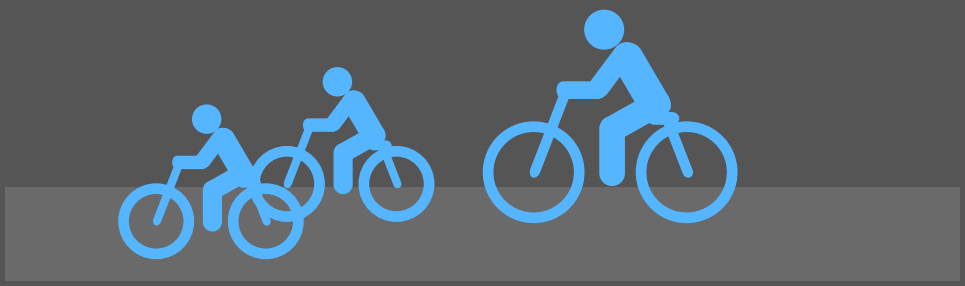
Last week it rained on Monday, Tuesday and Wednesday; it was sunny on Thursday and Friday – and then on Saturday and Sunday it was cloudy. Lucy went shopping on just one of the days last week and Sam went shopping on another day.

Here are a few facts :

- it was raining when Sam went shopping
- Lucy went shopping the day after Sam
- it wasn't raining when Lucy went shopping
- neither Lucy nor Sam ever go shopping at weekends

QUESTIONS : On which day did Lucy go shopping? And what about Sam ?

3 mean cyclists

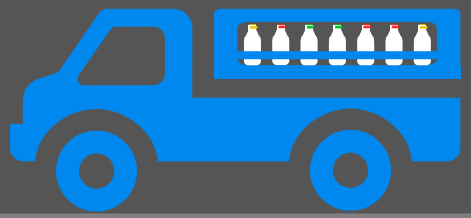


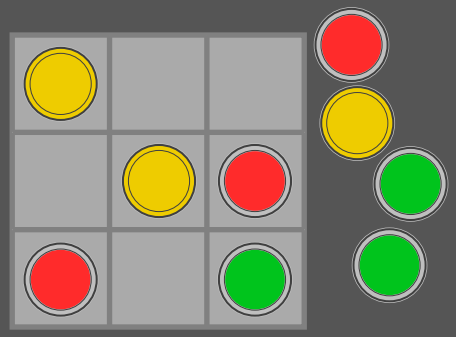
There are three children in the Evans family : older brother Roger and sisters Emily and Sarah. Here are some facts about their ages :

- Sisters Emily and Sarah are actually twins.
- The mean of all three children's ages is 7.
- The twins are 6 yrs old.

How old is Roger?

4 bottle-tops are go!





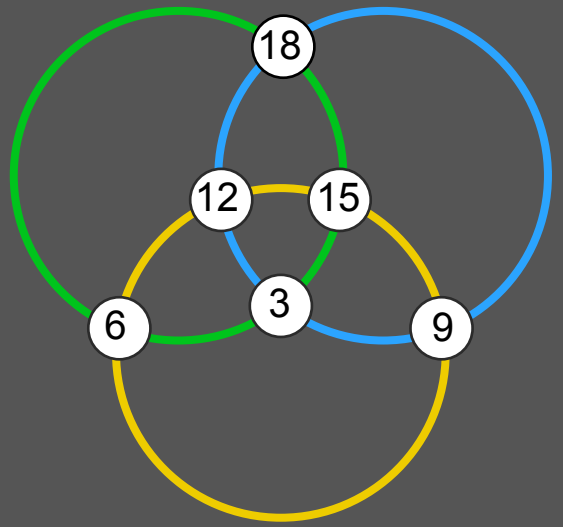
In some parts of Wales people still have their milk delivered each morning by the local milkman.The Morgan family (2 adults and 5 children) are just like this. Each day the milkman leaves them 9 bottles of milk : three are red-top bottles (ordinary milk), three are gold-top (extra-creamy milk) and three are green-top (low-fat milk).

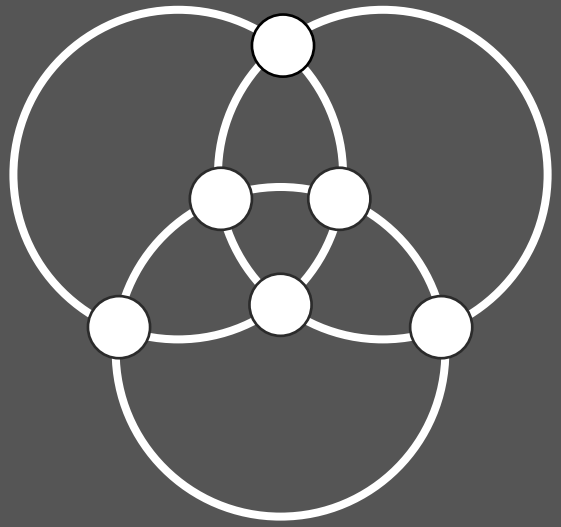
Here's a diagram showing you what the Morgans get each day. Can you arrange the bottles in the crate so that the coloured tops form a Latin Square? And, with these same colours, how many Latin Squares can you make?

You might not have come across Latin Squares before but the idea is a fairly easy one. A Latin Square is a square arrangement of things (numbers, colours, letters or whatever you like) with a simple rule : you can't have the same thing twice in any row or column. Latin Squares can be 3 x 3 or 4 x 4 or 5 x 5 – or any other size.

5 Japanese Magic Circles

Look at the diagram on the right. To start with, just look at the green circle. If you go round the green circle adding up the numbers which lie on it, you get 18 + 15 + 3 + 6, which equals 42. Now suppose you go round the blue circle adding up its numbers; you get the same total, 42. And with the yellow circle you get exactly the same total. Arrangements like this are called 'Japanese Magic Circles'.



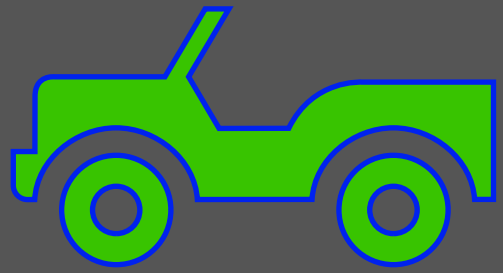


your problem : First of all, make a quick sketch of the diagram on the left : to do this, just draw a set of three overlapping circles (no need for colour) and draw little circles at all the places where the big circles cross. Next, find a way of arranging the numbers 13, 7, 4, 11, 8, 2 in the small circles so that you get the same total whichever big circle you travel round.

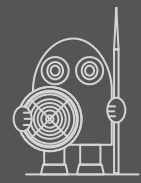
6 the job's yours !

‘The Major’ (a well-known criminal mastermind) is planning his next robbery. He’s got a team together but he’s still one man short. He needs someone who’s a good driver, can handle guns and is good at disguise. He has four men to choose from : Jake, Sam, Pete and Frankie. This is what we know about them :

- Jake and Sam are known to the police
- Jake, Sam and Frankie can drive
- Jake and Frankie are good safe-breakers
- all except Frankie can handle guns
- all except Jake are good at disguise
- both Jake and Pete know the area well



Which man will The Major choose to join his team?



7 spaced-out kids

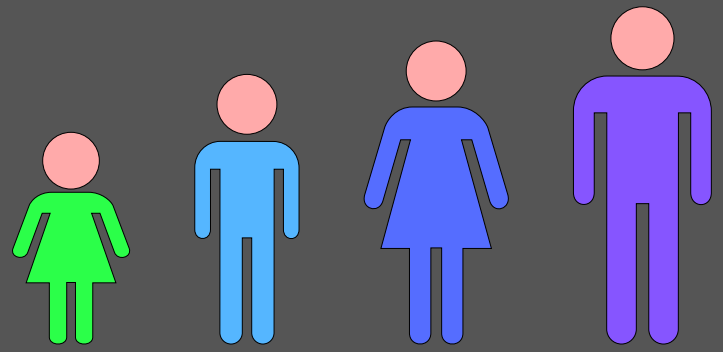
There are four children in the Pascal family; they are Anna, Bertrand, Christine and Daniel. Here are some facts abut their ages :

Anna is the youngest

Bertrand is 2 years older than Anna

Christine is 2 years older than Bertrand

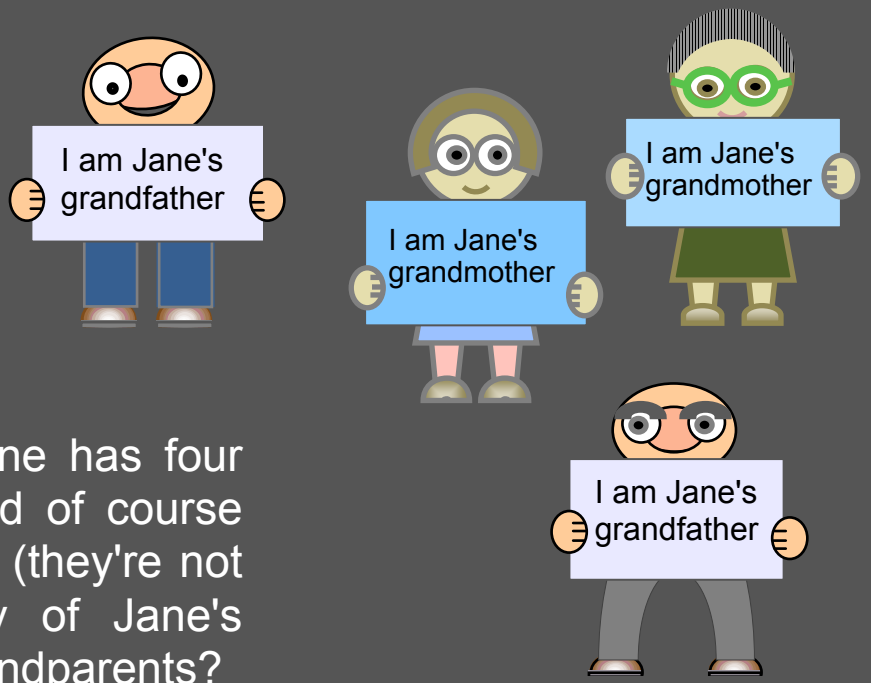
Daniel is 2 years older than Christine



If you add up the ages of all four children, you get a total of 28.

Your problem : how old is Bertrand?

8 Jane's people

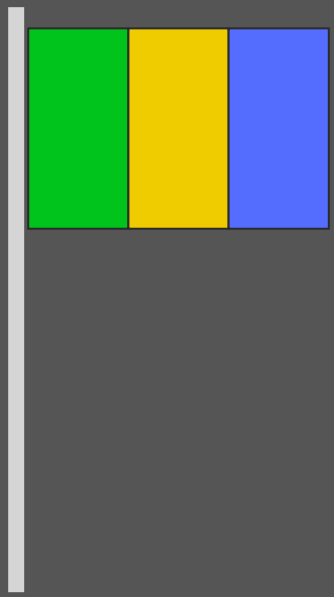


Jane and Mary are cousins. Jane has four grandparents (pictured here) and of course Mary also has two grandparents (they're not pictured here). But how many of Jane's grandparents are also Mary's grandparents?

9 flag day

One day, Mrs Robinson got her twenty-four Reception Class children to design flags. Her instructions were :

1. *Each flag must have three stripes.*
2. *You must use three different colours for your three stripes.*
3. *You must use blue, green and yellow as your three colours but you may arrange them in any order you like.*



On the right is a picture of one of the flags the children made. As you can see this one has green, yellow and then blue for its three stripes.

When the children had finished, Mrs Robinson saw that they had indeed made flags using different arrangements of the three colours.

- *How many different arrangements are possible, using each of the three colours once and once only ?*
- *If there were equal numbers of all possible arrangements, what's the probability that a flag picked up at random has a blue stripe between two other colours ?*

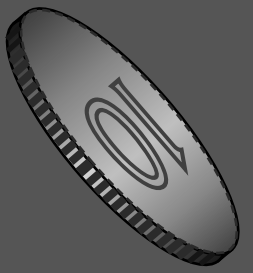
10 it's a clean sweep!

Billy and Joseph (his older brother) spend most of Saturday afternoon sweeping up leaves and generally tidying the garden for their neighbour, Dr Oliver. At the end of their labours, Dr Oliver pays them £15 for all they've done. Joseph says that as Billy is quite a lot younger than he is and also because Billy doesn't sweep up as many leaves, he won't share the £15 equally. Instead, says Joseph, he will be getting £3 more than Billy.

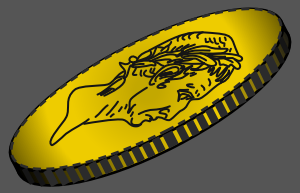
With this way of doing things, work out how much will be given to each of the boys.

heads and tails

Every coin has two sides : a head and a tail. Jake has an ordinary 10p coin in his pocket. He takes it out, flips it into the air and catches it on the palm of his hand. What's the probability that the coin has landed head-side up? As you probably know, the answer's 1/2. (That's because if you flip the coin often enough, it will land heads-up about half the time.)



Jake's sister Annabel has two older coins, a 10-cent coin (silver) and a 20-cent coin (bronze), both exactly the same shape, size and weight. She flips them both into the air together, then skilfully catches them in the palm of her hand.



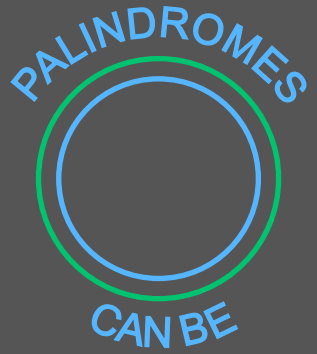
- What's the probability that when Annabel looks at the coins, she finds they're showing 2 heads?

- What's the probability that when Annabel looks at the coins, she finds they're showing a head and a tail?

NOTE : In case it isn't clear, the 'head' sides of Annabel's coins are the sides with the image of a head on them – and the 'tail' side of her coins are the sides with either a '10' or a '20' showing.

12 will there be time ?

You probably know what a palindrome is – it's something which reads the same from left to right as it does from right to left. In other words, you get the same result whether you go through the thing backwards or forwards.



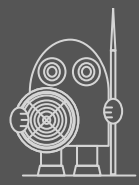
single words eg names like anna or otto

phrases eg never odd or even

whole sentences eg was it a car or a cat I saw?

numbers eg 2734372

Your problem : As you know, a digital clock always has four figures in the display. Now, if you look at the clock on the right, you'll see that the time it's showing make a palindrome. Try to find all the palindromes this clock will show in each 24-hour period.



13 mean Mr Francois

Mr Francois is a very angry man. In fact, last week the average (mean) for Mr Francois getting angry was 7 times per day. How do we know this figure? Well, last week some of his pupils noted each time he got angry and then at the end of the week they calculated the mean in the usual way. The figures we have for each day are as follows :

Monday: 4 times, Tuesday: 8 times, Wednesday: 7 times, Thursday: 6 times, Friday: ?

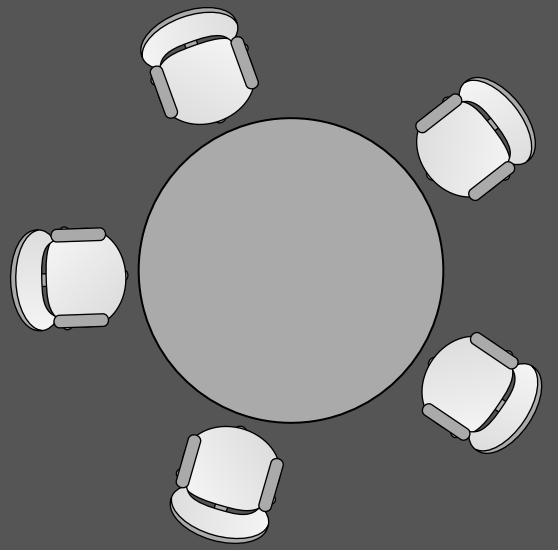
When Mr Francois gets angry his eyes bulge and his face turns a sort of dark red; he also shouts and waves his arms about rather a lot. It's a frightening thing to see. But the figure for Friday is missing from the table above. What should it be? How many times did Mr Francois get angry on Friday? That's your problem . . . anger is his problem.

ps He pronounces his name 'Frarn-swa'

14 come round for a meal !

Five friends – Alfred, Beatrice, Charles, Diana and Ellie – are having a meal together. Here's some information about exactly where they are sitting (it's a round table, by the way) :

- Beatrice is sitting between Charles and Diana
- Alfred is sitting on Ellie's right
- Diana is sitting on Alfred's right



Try to work out who is sitting where and then answer these two questions :

1 Who is sitting on Ellie's left?

2 Who is sitting on Diana's right?

15 digit-sums

What exactly are 'digit-sums'? Have a look at the mappings below and you'll soon get the idea :

421 –> 7

435 –> 12

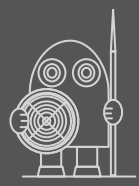
455 –> 14

470 –> 11

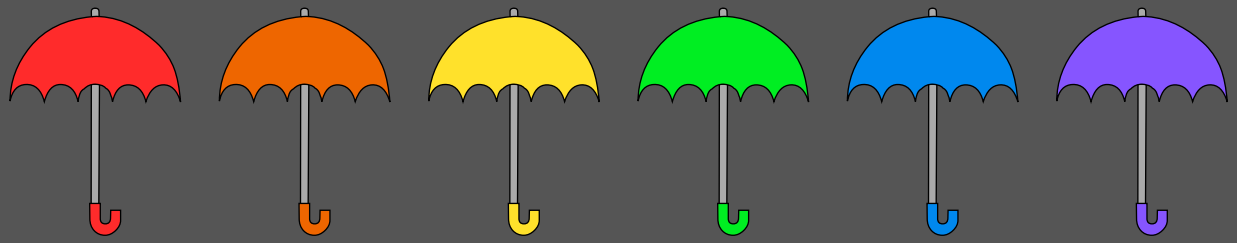
Can you find a simple connection between the three-digit numbers shown on the left and the numbers shown on the right ?

You've probably seen that if you add up the digits of any number on the left, this total (called the 'digit-sum') is what's shown on the right . . . Now answer these :

- what's the smallest digit-sum from numbers over 400 but under 500 ?
- what's the largest digit-sum from numbers over 400 but under 500 ?
- how many numbers over 400 but under 500 have a digit-sum of 12 ?



16 singin' in the rain



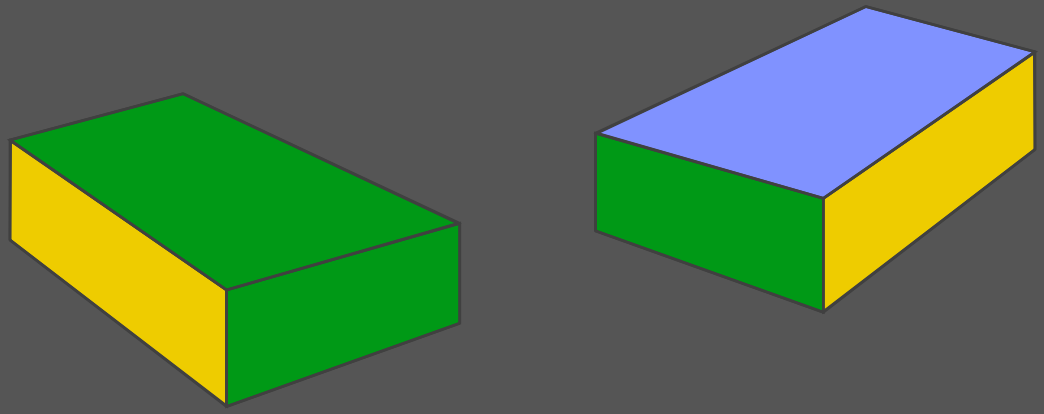
At the end of a very wet year, the Manhattan Umbrella Shop looked back at which umbrellas had sold well and which had sold poorly. They counted the sales of different colours and then they listed all the colours in order. Here's what they found :

- yellow and purple sales were equal
- green came above orange
- there were three colours between orange and blue
- there were 50 more yellow umbrellas sold than orange ones
- red topped the poll – more red umbrellas were sold than any other colour
- there were two colours between blue and green
- blue was a close runner-up; it almost came first

Using the information above, work out the positions of all six umbrellas in the final list.

17 no red faces here !

Here are two different views of the same box :



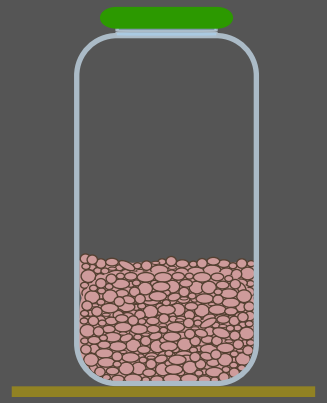
- there are two green faces
- only one of the two smallest faces is green
- the blue face you see doesn't have an edge in common with any other blue face

your question : considering the facts above and looking at the two views of the box, can you work out how many yellow faces there must be ?

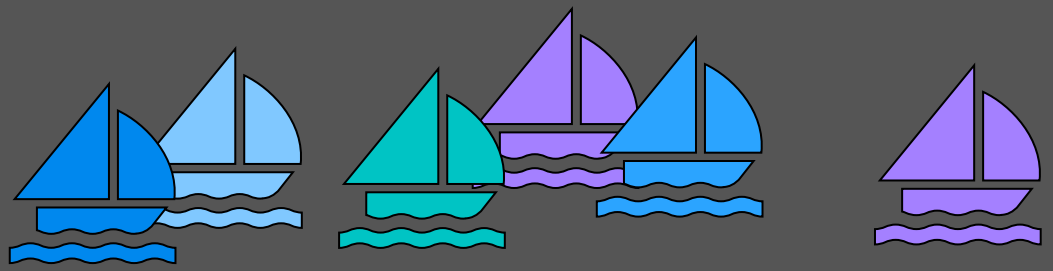
18 completely nuts !

On Boxing Day, there's a jar full of nuts next to the Christmas tree. The jar and the nuts together weigh 1.225 kilograms. A few days later, exactly half of the nuts have been eaten; now the jar and nuts together weigh just 784 grams. By New Year's Day the jar is empty.

What does the empty jar weigh ?



19 Annabelle sails



Annabelle loves sailing. Last weekend she took part in the annual Roxburgh Regatta in Nevada, USA, where the main event was a two-part race across Lake Hubron. On the Saturday, competitors had to sail straight across the lake from Roxburgh to Gadlas Creek – and on the Sunday, competitors had to sail the very same course in the opposite direction. Each day's race began at 11:00 exactly. The distance from Roxburgh to Gadlas Creek is exactly 18 kilometres, by the way.

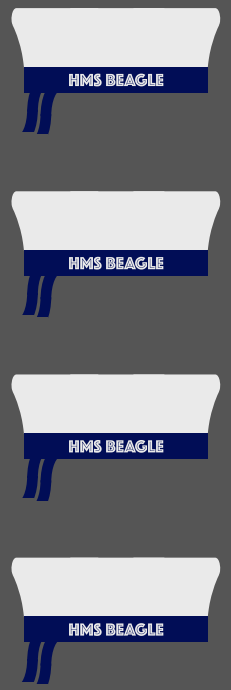
On the outward journey, Annabelle sailed past a small island. Next day, on the race back to Roxburgh, Annabelle sailed past this island once again. The strange thing was that she passed the island at exactly the same time each day. The other thing to report is that on the outward race, Annabelle sailed twice as fast as she did on the return race. (Probably something to do with the prevailing wind.)

How far is the small island from Roxburgh?

20 dockyards & warships

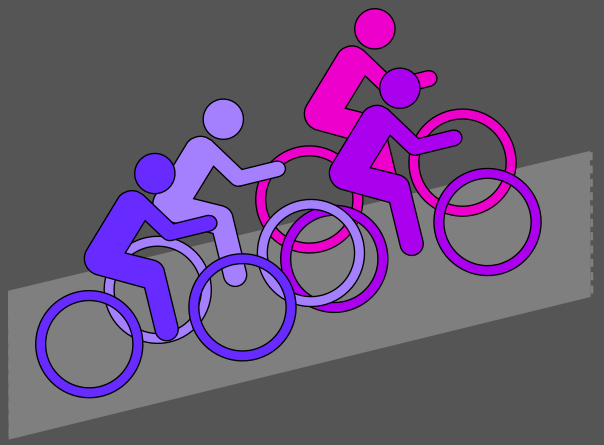
One day in Spring, a small naval patrol vessel, the HMS Beagle, sets off from Plymouth. It's bound for Rosyth in Scotland and the four sailors on board are looking forward to the trip. The names of the sailors are Anson, Byng, Cavendish and Douglas. There are various things on the vessel which must be attended to at all times, such as steering, keeping a lookout, maintaining signals and so on. To do these things properly, there must be three sailors on deck at any one time.

Your problem is this : How many different groups of three can you make up from these four sailors?



21 uphill & downhill

The Four Peaks Cycle Race takes place on the first Sunday in May each year. This is one of the most famous outdoor cycle races in the country and hundreds of brave cyclists, both men and women, take part. It's a difficult race, featuring some really tough uphill stretches and some quite giddy downhill ones. (There are no level stretches.) Here's the official description of the course :



"THE COURSE : The race begins at Belvoir Point (743 metres above sea-level) and from here climbs steeply up to Carr's Peak (height 982 m), before a shorter section takes the riders down to Denton Dene (height 823 m). Next follows the longest climb of the whole race, which takes the riders up to Eagles' Nest (height 1253 m). After this come short downhill and uphill sections, taking the riders first to Fallon Peak (height 1107 m) and then to Glyn Mount (height 1261 m). Next comes the longest downhill section of the race, which leads from Glyn Mount to Home Hollow (height 475 m). The race ends with a slightly easier climb to Ingle Peak (height 687 m) and a short downhill descent to the finish at Juniper Road (height 521 m)."

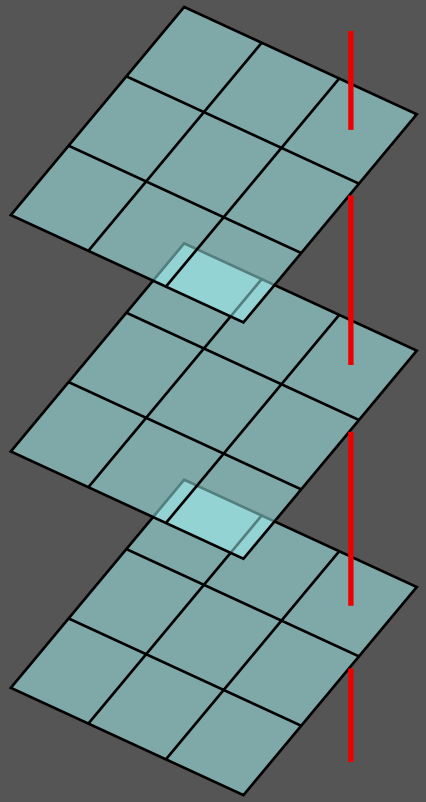
YOUR QUESTION

As one happy cyclist put it,'There's a good deal more going down than climbing in this race'. Exactly how many metres more going down is there than climbing ?

22 latin cube

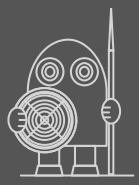
You probably know what a Latin Square is : it's a square arrangement, 3 x 3 or perhaps 4 x 4 or 5 x 5, with numbers or letters or colours or shapes placed so that you never get the same thing twice in any row or column (though you are allowed the same thing repeated in a diagonal). Here's an example of a Latin Square :

1	2	3
2	3	1
3	1	2



Your problem is to place the numbers 1,2,3 on the three grids so that you never get the same number twice in the same row or column horizontally or vertically. For example, if you go down the red column you shouldn't meet the same number twice. . .

Think of the three square grids (pictured here on the left) as the three floors of a cubical building . . .



23 just two gorillas

In a Gorilla Rescue Centre in Eastern Congo (Africa), there's one enclosure with just two residents – a couple of young orphan gorillas called Sultan and Solomon. These two gorillas were rescued from poachers and they're now being cared for by a dedicated wildlife team. This team will also prepare the gorillas to go back to the jungle where they belong.

Sultan

For the time being, the gorillas live on a special mix of food and sadly it's running out : there's just a certain number of sacks left and the team can't get hold of any more at present. So, that's a real problem for the team. Now here's your problem :

If they had only Sultan to feed, the remaining sacks would last him just 10 days. On the other hand, if they had only Solomon to feed, the remaining sacks would last him exactly 15 days. But of course both Sultan and Solomon have to be fed. Try to work out how long the remaining sacks will last . . .

21 consecutive numbers . . .

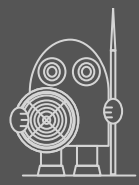
You know what *consecutive numbers* are – they're just numbers which follow each other. For example : 9, 10 are consecutive numbers and so are 11, 12, 13.

You can get various totals by adding together *pairs* of consecutive numbers; for example, 3+4=7 and 8+9=1 . . . Or you can add together sets of *three* consecutive numbers; for example, 5+6+7=18 and 9+10+11=30 . . . Or you can add together sets of *four* consecutive numbers, or sets of *five* consecutive numbers and so on.

a. You can get a total of 9 in two different ways : either from a *pair* of consecutive numbers (4 + 5) or from *three* consecutive numbers (2 + 3 + 4). Can you find another number under 20 where the same thing is possible?

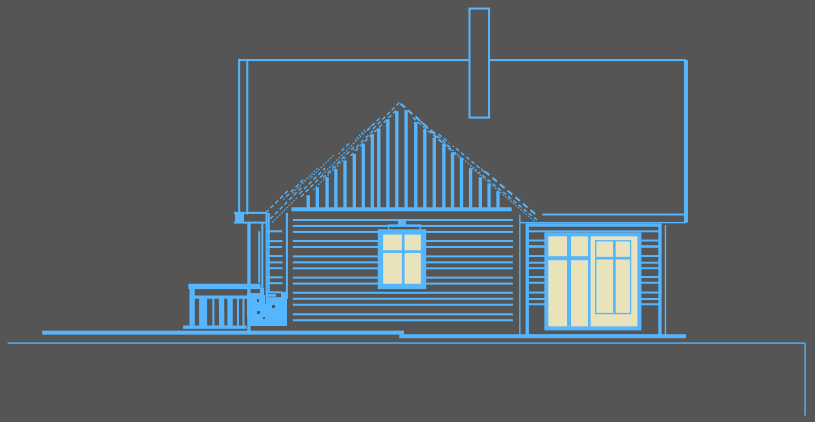
b. You can get a total of 18 in two different ways : either from *three* consecutive numbers (5 + 6 + 7) or from *four* consecutive numbers (3 + 4 + 5 + 6). Can you find another number under 40 where the same thing is possible?

c. Can you find a number under 50 which can be made either by adding *two* consecutive numbers or by adding *four* consecutive numbers?



25 the little house on the cliff . . .

John and his wife Amina used to own a small house on the edge of the cliffs somewhere near Newhaven. They bought the house for £100,000 but the following year there were newspaper reports that some cliffs in the area were unstable. The value of the little house fell by 20%. But then a new geological survey showed that their house was in fact built on solid ground. The value of the house went back up again to £100,000.



Exactly what was the percentage increase when the house's value returned to its original £100,000? (nb The answer is not 20%!)

26 sum of three primes

prime numbers

When Sally and her friends go into the maths room, Mr Pascal, the maths master, has taped a piece of paper onto the whiteboard. He tells the class that underneath the paper there are three prime numbers written on the board – and not just any three prime numbers chosen at random . . . these three prime numbers add up to 100.

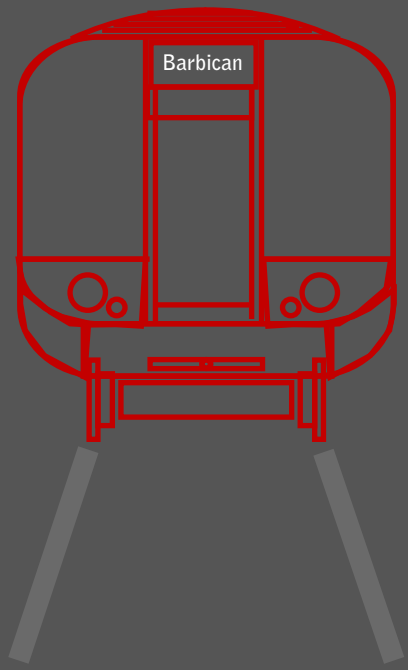
'What's the question?,' ask the pupils,'What is it you want us to work out?'

'The question is simply this,' says Mr Pascal,'What's the smallest of these three numbers?'

Remember : a prime number is a whole number with exactly 2 factors (itself and 1).

27 transport for London

120 people who work for Williams' Bank in the City of London were asked how they travelled to work. Altogether 82 reported that they travelled by train and altogether 45 reported that they travelled by tube. 10 said they didn't use either the train or the tube. How many used both the train and the tube to get to work?

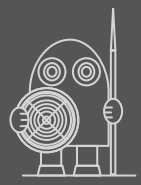


28 marking time . . .

Here are Anthony's marks for the end-of-year exams. There were exams in six subjects – and those six teachers chose six different ways of marking, as you can see here. Because of this, Mr Barnes (Anthony's form teacher) is in trouble. He needs to work out an average mark for each boy and he just doesn't have the faintest idea of how to go about this.

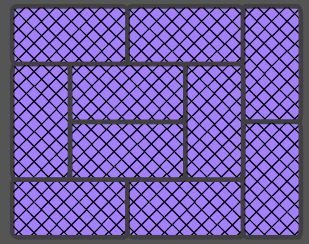
Take a look at Anthony's marks and then see if you can find a sensible way of working out his average.

English	36 / 60
French	48 / 50
Maths	74%
Science	18 / 20
History	30 / 75
Geography	24 / 40

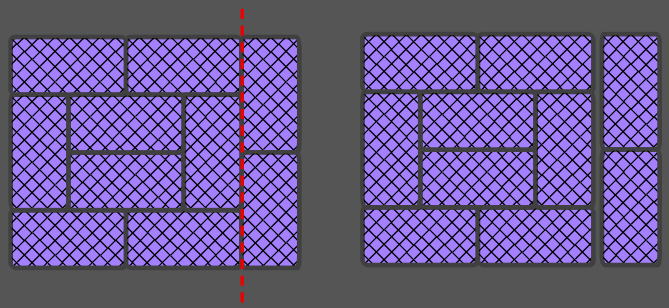


29 fault lines

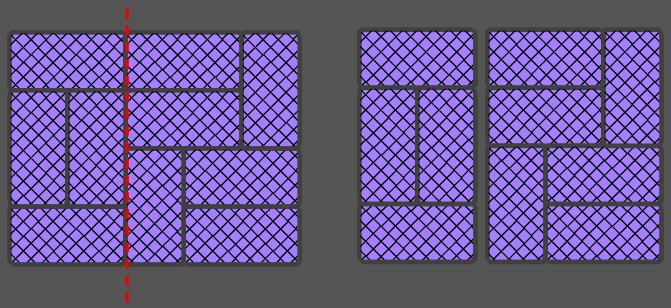
Suppose you have a number of 2 x 1 tiles. If you had a 5 x 4 rectangle, could you tile it completely with your 2 x 1 tiles? Well, the answer is yes – and in more ways than one. Here's one possible way :



But one thing we notice about this tiling is that it has a fault line, as you can see here :



And here's another example of a 5 x 4 rectangle tiling – and this also has a fault line :

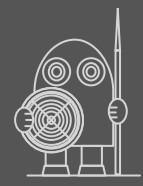


** a 'fault line' is a line which runs right across the rectangle from one side to the other, in effect dividing the rectangle into two parts*

In fact, it's impossible to tile a 5 x 4 rectangle with 2 x 1 tiles without getting a fault line somewhere. However it is possible to tile a 6 x 5 rectangle completely using 2 x 1 tiles and not to get a fault line.

- Can you work out a way of doing it?

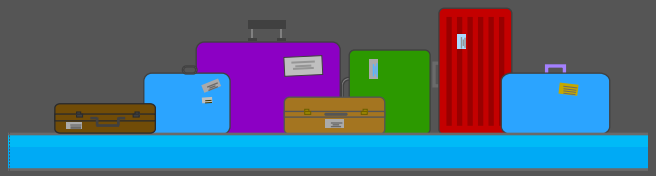
** You'll find dominoes, face down, make quite good plain 2 x 1 tiles if you're looking for a practical way of trying things out.*



30 an open and shut case

The Johnsons are going on holiday. Whenever they go away, the Johnsons always take lots of luggage with them. Mr Johnson is the worst culprit by far, as you can see from this information about their cases (to keep things shorter, we're calling them Father, Mother and Billy) :

- Father's luggage weighs twice as much as Mother's luggage.
- Billy's luggage weighs only half as much as Mother's luggage.
- The difference in weight between Father's luggage and Billy's luggage is exactly 150 kg.

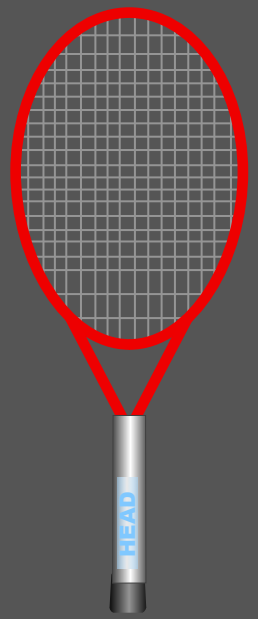


So, how much does Mother's luggage weigh?

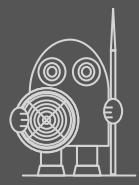
31 what a racket!

Imagine you're organising a tennis tournament for your tennis club. Suppose by chance 16 people enter. You could start off with 8 matches, which would produce 8 winners. Then you could have 4 matches and so these 8 would be reduced to 4. Then 2 more matches (the 'semi-finals') would give you 2 winners. And these 2 could take part in the final match to produce 1 final winner, the 'champion'. It all works out easily, doesn't it?

Now suppose the following year only 14 players turn up. You would have liked 16 but you have only 14, so you give 2 players a 'bye' (that means they go straight through to the next round). Now you get the remaining 12 to play in 6 matches, giving you 6 winners. This means you have 8 players left (6 winners from matches they've played and 2 people who've had a 'bye'). With 8 players remaining, the rest is easy . . .



Now imagine the following year is a special Jubilee Year for your tennis club and you advertise the tournament in several places. To your surprise, 29 players enter. How many matches altogether will have to be played before you can end up with just one 'champion'?



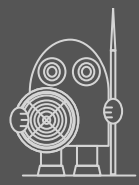
32 late for work !

Mr Brodie was Chief Clerk in the Accounts Office of a small factory, where he was in charge of eight junior clerks. These clerks really did all the work (Mr Brodie just made sure they kept at it). The junior clerks were supposed to arrive at 7:30 each morning to start their day's work but in fact they turned up at all sorts of times. Mr Brodie was getting angrier and angrier about this until one morning he made a list of when each of the junior clerks arrived. This is the list :

Albert	*7:20*	*Ellie*	*7:36*
Ben	*7:35*	*Fred*	*7:27*
Carrie	*7:26*	*George*	*7:37*
Daisy	*7:22*	*Hattie*	*7:29*

Mr Brodie looked at his list and straight away he began to shout at the clerks, saying were not being punctual . . . but then one of the clerks spoke up and said, 'Well, on average our times are not too bad!'

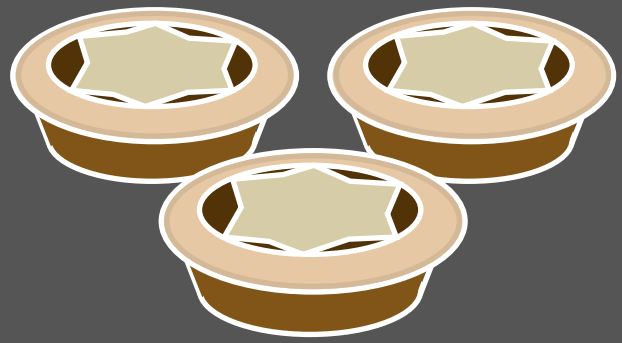
What was the **mean** (average) arrival time of the eight clerks?



33 tickets and pies

1 Last week, John went to a concert. On the way in, he bought a ticket for the concert and also a programme; altogether these two things cost him £11. In fact, the ticket cost £10 more than the programme. Don't do any working-out on paper – just think for a moment and then write down how much you think John paid for the programme and how much he paid for the ticket.

2 Sisters Sue and Jenny spent Sunday afternoon baking mince pies. When they had finished, there were 47 mince pies altogether. Some mince pies were for their own family and some were for the School Fair. Both girls agreed that the School should have 13 more mince pies than the family. So, how many mince pies did the family end up with?

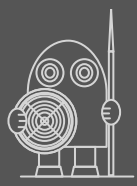


34 a tricky question

It was 11:40 and Mr Pascal, the maths teacher, was sitting in the staff room at Low Moss School with a pile of books in front of him. He'd just finished marking the books and he was feeling rather pleased with himself. He looked up at the clock, saw that it was almost 11:40 and remembered that as it was the last lesson of the morning and as it was also a thursday, he should head for class 3N. He enjoyed teaching this class and they enjoyed his maths lessons.

As Mr Pascal entered the 3N classroom, he saw at a glance that the pupils were all there. Like many of the classes, 3N had an equal number of boys and girls. Mr Pascal asked the class a tricky question to start the lesson and within a minute or two, four girls had their hands up; the boys were still thinking and all had their hands down. At this moment there were one and a half times as many boys as girls with their hands down.

So, how many pupils altogether were there in 3N?



35 6-a-side football

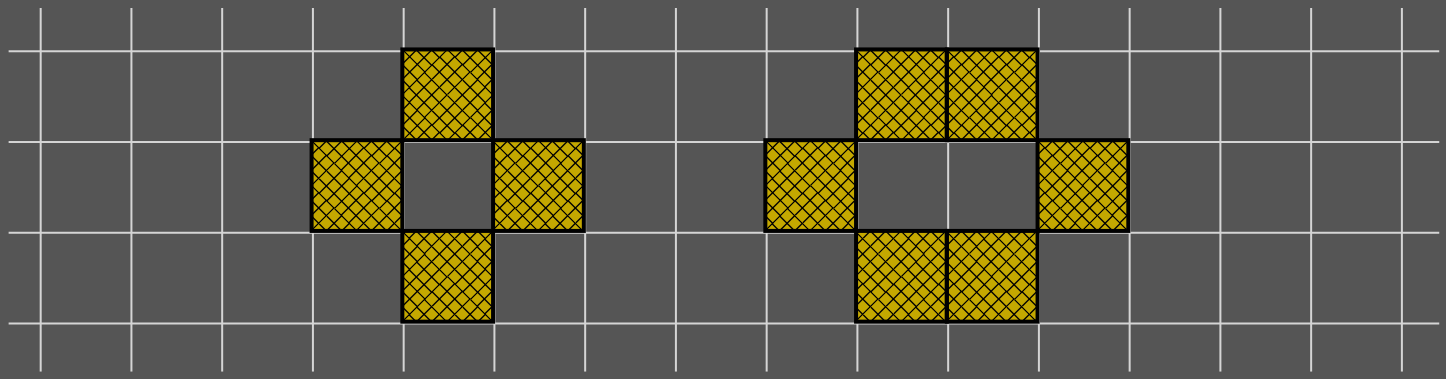
One Saturday morning, Jimmy and Hassan went down to their local gym for a six-a-side football game. Hassan's got a sister Amina who's a bit of a maths genius and when he arrived home, she asked him who had been playing for his team. 'Well, there were six of us down there', said Hassan, 'but I won't tell you their names. Instead I'll give you some info about their shirt-numbers!' First of all, Hassan said that obviously the six numbers were all different. Then, thinking of the numbers in a list, smallest first, he gave her these facts:

- The second number is 3 and the fifth number is 22
- If you add the first four numbers, you'll be just 1 short of the sixth
- If you square the second number, you'll get the third
- The fifth number is 5 less than the sixth
- The first three numbers add up to 13

What were the six numbers the boys' team had on their shirts ?

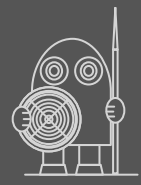
36 sheep may safely graze

On Tudor's farm, they use a 1 metre square grid (painted on the farmyard floor) and some bales of hay (they're one–metre cubes) to make sheep pens. When you're making one of these sheep pens, the bales of hay may just touch at the corners or they may lie exactly side by side – but there must be no gaps! Here are two examples of sheep pens from Tudor's farm :



As you can see, with 4 bales of hay you can enclose an area of 1 square metre – and with 6 bales you can enclose an area of 2 square metres.

What's the maximum area you can enclose if you've got 9 bales of hay?



37 Pablo's progress

PABLO PABLO PABLO PABLO

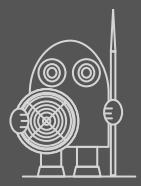
Summer-time! And once more the end-of-year exams are here. Pablo doesn't mind the maths exam because maths is something he enjoys. When all the exams are over and the marks come back, Pablo is pleased to find out that once again he's done fairly well on the maths exam.

For your information, this is how the maths paper is organised : the Mental Maths section is marked out of 20, the Calculations section is marked out of 30 and the Problems section is marked out of 50. So obviously if you have your actual marks for the three sections, you could just add them up and you'd get a mark out of 100, (which would, of course be your percentage).

But Pablo's teacher, Mr Ponticello, gives him his marks for the three sections like this :

Mental maths 85% Calculations 90% Problems 86%

Mr Ponticello then challenges Pablo to work out his overall percentage for the whole paper. Pablo is baffled – but not for long! Can you work out Pablo's final percentage ?



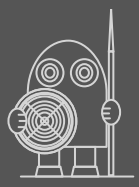
38 happy birthday James !

Sophie and her brother James have the same birthday. But they're not twins! Sophie is actually older than James; it just happens that their birthdays fall on exactly the same day. Here are two facts about their ages :

- Next year, Sophie's age will be three times James' age.

- Last year, Sophie's age was four times James' age.

From these two facts, work out how old James will be next year. Use any method you like to get an answer.



39 new year neighbours

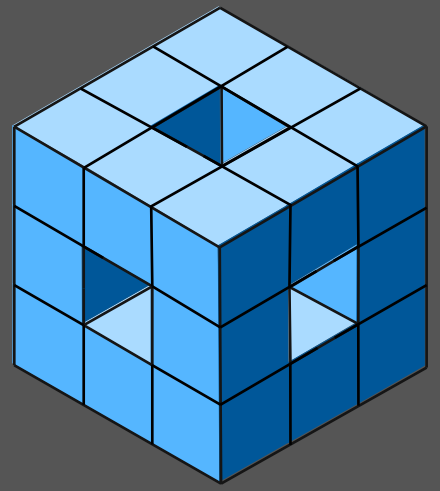
Along Riverside Walk all the houses face the river and so they're numbered consecutively : 1, 2, 3, 4, 5 . . . and so on, up to 52. The Smart family move into number 10 Riverside Walk on 1st Jan. and young Alec points out that the house number in their address is rather special – they live at number 10 Riverside Walk – and 10 just happens to lie between a square number (that's 9) and a prime number (that's 11).

How many special addresses like this are there along Riverside Walk, with a prime number on one side and a square number on the other? Try to list them all.

optional extra : There's only one odd number among the answers. Why is this? (Hint : ask yourself whether there's anything different about the prime neighbour of the one odd answer.)

40 the heartless cube

Amy makes this shape by sticking together a number of 1cm wooden cubes. Her new shape is a 3cm x 3cm x 3cm cube – but with each face having a 1cm square hole going all the way through to the opposite face.



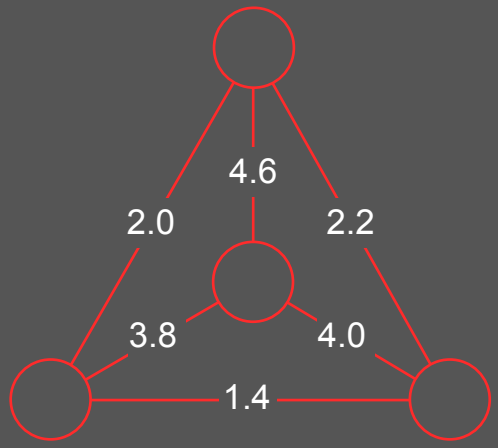
- How many 1cm cubes are there in the shape Amy has made?

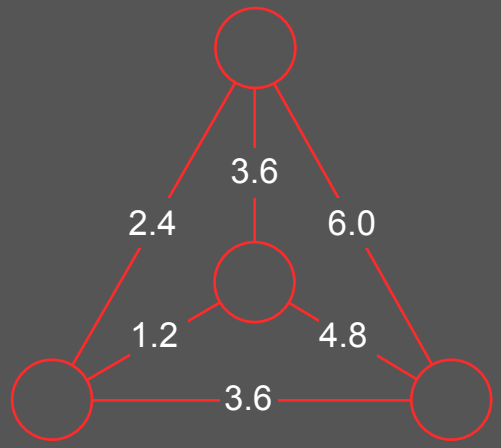
- Amy's shape is made of just plain wood but – she has some 1cm squares of sticky paper in all sorts of colours. She decides to cover every part of her shape (that's to say, every part which is open to the air), using different kinds of blue. How many of these 1cm sticky squares will Amy use? (Or, to put the question a different way, what's the total surface area of Amy's shape?)

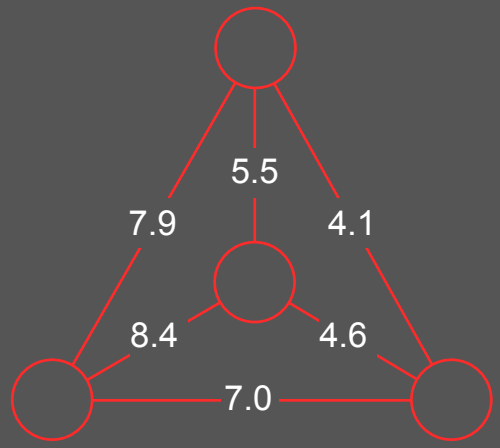
41 more number triangles

Now number triangles just got a whole lot harder ! You've come across these before and you probably remember how a number triangle works : pick any two circles and the number between them is what the numbers in the two circles add up to. However – we've blanked out the numbers in the circles and you have to figure out what the originals must have been.

You've done this before with easier numbers and perhaps then you found a way of getting started and of working through to an answer. That's what you need to do in order to solve these three problems.



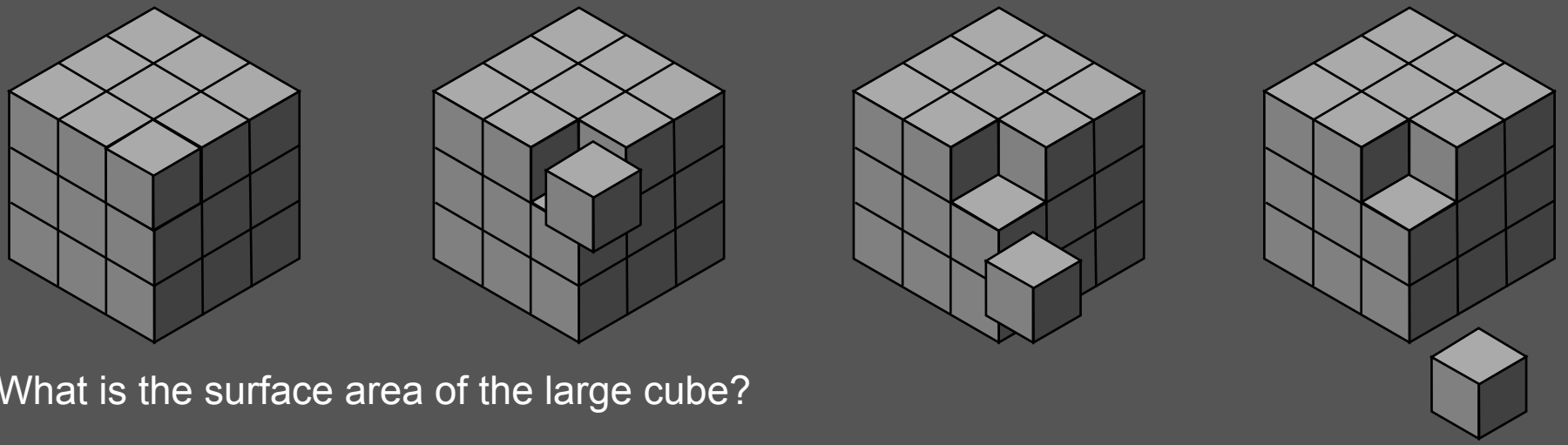




42 the missing cube

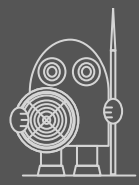
The large cube below (left) is made up of smaller 1cm cubes. You probably know what we mean by the *volume* of this large cube : it's just how many 1cm cubes it contains (27 here).

Surface area is a different idea : To work out a shape's surface area, you just find the area of each face of the shape – and then add all these areas together. As a cube has six identical faces, there will be six areas (all the same) to be added together. Here are two surface area problems for you to work out :



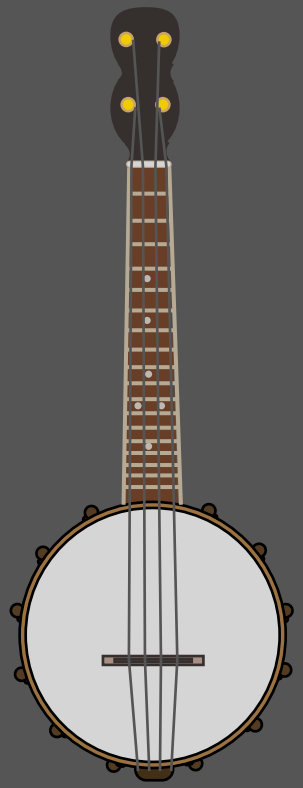
- What is the surface area of the large cube?

- The large cube has not been glued together very well and one afternoon one of the small corner cubes comes loose and eventually falls off (see pictures 2, 3 and 4 above). So now we have a large cube with one missing corner, together with a small cube. What's the total surface area of the large cube now it has a corner missing?



43 the frog and banjo

A few years ago there was a very successful pop group called '*The Amphibious String Band*' (you know what *amphibious* means). The group had a frog, two newts (called Newt 1 and Newt 2) and a toad; they mostly played country-and-western music. Because the frog was really the star of the group, they later changed their name to '*The Frog and Banjo*'. Each group member played only one instrument; here's some information about who played what :



- Newt 1 did not play double-bass
- Toad didn't play either lead guitar or rhythm guitar
- Frog played the banjo (pictured here)
- Newt 1 did not play rhythm guitar
- Newt 2 didn't play lead guitar

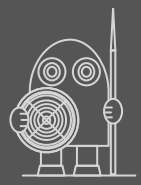
Can you work out who played which instrument?

44 cube calendar - months

You might remember book 1 problem 42 'calendar cube - days'. That problem was about a daily calendar made of wooden cubes. The numerals on the first two cubes showed the day of the month – and the letters on the remaining three cubes told you which month it was. The calendar was made by Mr Pascal, a maths teacher, using five plain wooden cubes and some white stick-on pvc numerals and letters.

In book 1 problem 42 you had to find a way of sticking numerals onto the first two cubes so that all dates in the month (that's to say, all numbers from 1 to 31) could be shown. The problem here is a rather harder one. Your challenge now is to think of a way of sticking letters onto three cubes so that by lining up the cubes in different ways you can show any of the twelve months from 'jan' to 'dec'. There are various ways of going about this but it's not an easy problem.

hint : If you need to, you're allowed to use the 'n' to also stand for 'u' – and you're allowed to use the 'd' to also stand for 'p'. (With both the 'n' and the 'd', turning the cube upside-down will allow the one letter to look like the other. And you'll find that in most sets of self-adhesive numerals you can buy, the 'n' and the 'u' are exactly the same shape as each other, as are the 'd' and the 'p'.)



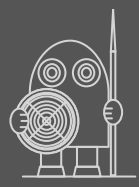
45 stairway to heaven

Soldier ants guard their colony and as you might guess, they like to keep fit. Milit is an ant just like this. To keep himself in trim he sets off every morning at the bottom of the church tower and he makes his way to the top by starting on step 1 and then jumping to step 4, then to step 7, then to step 10, then to step 13 and so on . . .

One morning, just as Milit starts to jump from step 1, a rather fat frog jumps from the top step (step 161); the frog then jumps onto step 157, then onto step 153, then onto step 149 and so on . . .

Both the frog and the ant jump exactly in time with the regular chiming of the church clock. This means that you have ant on step 1 at the same time as frog on step 161, then ant landing on step 4 just as frog is landing on step 157 and so on . . .

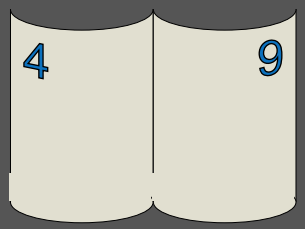
If the frog lands on a step at the same time as Milit the ant, then Milit will probably get squashed and die. Try to work out whether this terrible thing will really happen.



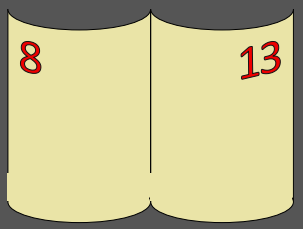
46 after the flood

It's that time of year and once again floods have hit the little Cornish port of Fowey. Waters swept through the printer's shop and many of the leaflets he had prepared for his customers simply floated away.

Jenny found one loose sheet from a booklet floating in her garden; she wasn't sure what the booklet was about but the page was A4 size with a fold in the centre – clearly it had originally been part of an A5 booklet. The loose page she had was numbered 4 and 9 on the one side, and 3 and 10 on the other side. How many pages were there in the original booklet ? And how many single A4 sheets had been used to make the booklet ?



Roger came across a single sheet from a completely different booklet. Again, the page had been part of a small A5 booklet. The page-numbers on one side of the sheet were 8 and 13 and on the other side of the sheet the page-numbers were 7 and 14. How many pages were there this time in the original booklet? And how many single A4 sheets had been used in making the booklet ?

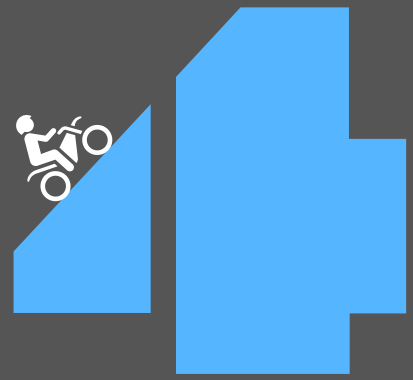


47 action fractions

Here's an unusual problem involving fractions :

What what number can you subtract from 4 and get the same result as when you multiply it by 4 ?

This number, as you might guess, is a fraction and because it's a fraction which can do two things, we call it an 'action fraction' : take it away from 4 and you get a certain answer; or multiply it by 4 and you get exactly the same answer. See whether you can find the action fraction we're looking for here.



USEFUL HINT : the action fracton we're after here is less than 1 . . .

48 trip saver

Every year at the start of October, Meriden High School runs a trip to London for 6th-form pupils. As well as various visits to well-known London places, the trip involves a 1-night stay in a Kensington Hotel. As with everything in life, there's a price to pay; in this case the overall cost of the trip is £80 per pupil. Both Khori and his sister Liss want to go on the London trip; but for each of them this will have to involve some careful saving-up. Here's how it goes :

LONDON BRIDGE
TRAFALGAR SQUARE
MAIDA VALE ABBEY ROAD
ROYAL ALBERT HALL
KNIGHTSBRIDGE
WATERLOO KINGS CROSS
MARBLE ARCH SOHO
ST. PAUL'S CATHEDRAL
PICCADILLY CIRCUS
WESTMINSTER ABBEY
OXFORD CIRCUS

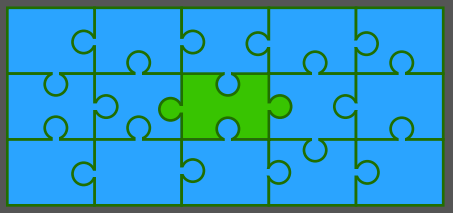
Liss says there's no time like the present and so she begins to save in the first week of the year; let's call it week 1. Liss handles her spending-money carefully and she saves £2 each week without fail. Khori says he needs all his spending-money, so he doesn't start saving for the trip until he's found a part-time job delivering leaflets. Each week then, starting in week 9, Khori does his few hours of paid work and each week from week 9 onwards he saves £3.00 towards the trip. He's started later than his sister but he's saving faster!

By the way, all money for the trip must be paid for by the end of week 40.

One week Khori and Liss find that they have paid exactly the same amount.
(a) In which week of the year does this happen? (b) What is the amount?

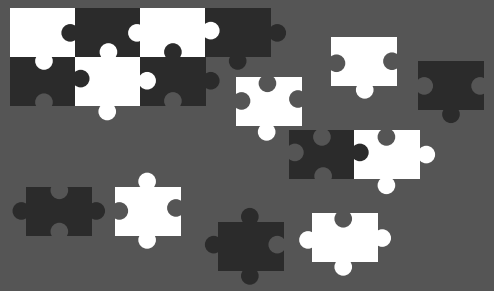
49 black & white jigsaw . . .

Here's a picture of a very simple jigsaw (to make it easier to picture, we've made the main inner edges straight). As you can see, there's one piece, coloured green here, which you could call the 'central piece' – meaning that it has the same number of pieces above and below it **and** the same number of pieces to the left and to the right.



Perhaps you remember an earlier problem about Syed and his jigsaw of 1458 pieces. That jigsaw had 54 pieces along each longer side and 27 pieces along each shorter side. Did Syed's jigsaw have a 'central piece'?

Next, once again thinking of Syed's jigsaw and its 1458 pieces : Suppose the pieces of this jigsaw were coloured black and white in such a way that you could lay out all the pieces and never have a black edge next to a black edge or a white edge next to a white edge. Would there be exactly the same number of black and white pieces?



50 match days count

You know how it is with a football league - there are days when all the clubs play and there are days when only some of the clubs play (and the others have a rest day). Of course, how easy (or how hard) it is for the organisers to work out a programme of match-days depends on how many teams there are in the league.

In Hythe there are just 4 teams who compete every Autumn half-term for the Hythe Schools Cup. The competition is run as a league – that's to say, every school must play every other school and points are awarded for either a win or a draw. All the matches take place in the Hythe Town Stadium, so there's no need for 'home' and 'away' matches. And of course, no team is ever expected to play twice on any day.

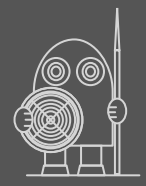
As you can probably see, 6 matches altogether must be played to complete the competition. It's easy for the organisers to fit these 6 matches into 3 match days (with no need for any rest-days!). The chart on the right shows one way of arranging things; as you can see, it shows team A playing team B and also team C playing team D on the first day – then on the second day, team A plays team D and team B plays team C – and on the third day, teams B and D play one match whilst teams A and C play another.

4 teams

	resting
A B / C D	–
A D / B C	–
B D / A C	–

= 3 match-days

Problem : What's the smallest number of match-days you would need if another school joined in the competition (making 5 schools in all) ?

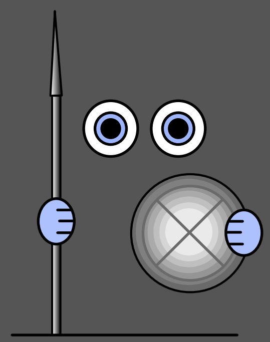


no

problem !

book ?

problem book



four winds