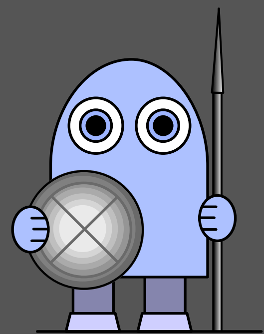
no problem !

book 2

answer book

four winds



mark time . . .

This looks like the sort of problem where *experiment* might well be the best approach. Let's try some different ages for Mark and see what happens :

Susan	Mark	Susan	Mark	Susan	Mark
3	1	6	2	9	3
4	2	7	3	10	4
5	3	8	4	11	5
6	4	9	5	12	6
7	5	10	6	13	7
8	6	11	7	14	8

To begin with, we've just worked out what happens when Mark is 1 or 2 or 3. As you've probably worked out by looking at our lists, the two red lines link the children's ages now and in two years time.

It doesn't take long to see that if we have Mark as 2 years old now, then everything works out fine. Reading from the middle set of results, we can see that our answer is : in two years' time, Susan will be 8 and Mark will be 4.

PTO

mark time . . .

ps. You might have realised at the beginning that Mark's age now must be an even number. This makes life a lot easier, as it halves the number of ages we need to try out for Mark. Why must Mark's age today be an even number? Here's the thinking behind the idea . . .

1. In two years' time, Susan's age will be exactly double Mark's age . . .

2. Which means that in two years' time, Susan's age will be an even number . . .

3. So Susan's age must be an even number now (now differs from then by exactly two years) . . .

4. And so Mark's age must be an even number now (because if it were odd, then three times it would give an odd number for Susan's age) . . .

ans 2 shopping day

One way of setting about this problem is to draw up a grid, like the one on the right, showing the days of the week in one direction and the names of the shoppers in the other direction :

	M	T	W	Th	F	Sa	Su
Sam							
Lucy							

Next, we can add symbols showing what we're told about the weather on different days :

	M	T	W	Th	F	Sa	Su
Sam							
Lucy							

PTO

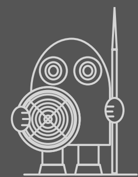
ans 2 shopping day

Now we look at the facts we're given about Sam and for each thing we're told, we put a cross in every square where we know he can't have been shopping. (For example, fact 1 tells us that, 'It was raining when Sam went shopping'. So we can put a cross for him under all the dry days.) Then we look at the facts for Lucy and put crosses wherever she can't have been shopping.

	M	T	W	Th	F	Sa	Su
Sam				X	X	X	X
Lucy	X	X	X			X	X

Taking a hard look at the table we've drawn up, we can see the answer to our problem. We're told that Lucy went shopping the day after Sam and, as you can see, the only days when this happened were Wednesday and Thursday. So there we have it :
Lucy went shopping on Thursday; Sam went shopping on Wednesday.

	M	T	W	Th	F	Sa	Su
Sam			✓	X	X	X	X
Lucy	X	X	X	✓		X	X



ans 3 mean cyclists

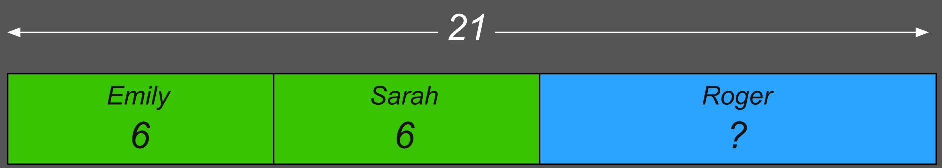
This question is about means, so stop for a minute and think about how you work out means : you add up a set of numbers and then you divide by how many numbers there are. That's easy and you've probably done it lots of times. If someone tells you that five numbers add up to 20, you can easily work out the mean of the numbers : it's just 20 divided by 5, that's to say 4.

Now imagine someone tells you that they're thinking of a different set of numbers; this time there are eight numbers and their mean is 6. Can you work out what the numbers must add up to? If you're stuck, just stop and think how the mean was worked out in the first place : the total was divided by 8. The only number you can divide by 8 and end up with 6 is of course 48. So the total of these eight numbers must be 48.

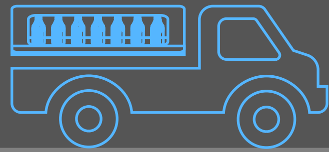
Let's take another look at our question. We have three numbers (ages in this case) and their mean is 7. What must their total be? Answer : 21

This total of 21 is made up of two lots of 6 (the twins' ages) plus another number (Roger's age). This other number has to be 9 (that's 21 – 12). So Roger is 9.

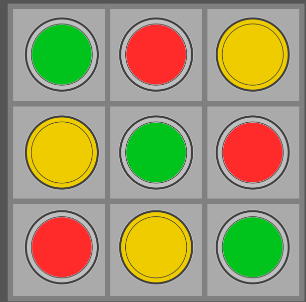
Here's a diagram to illustrate :



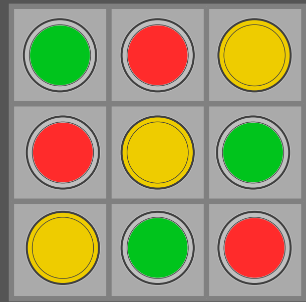
ans 4 bottle-tops are go!



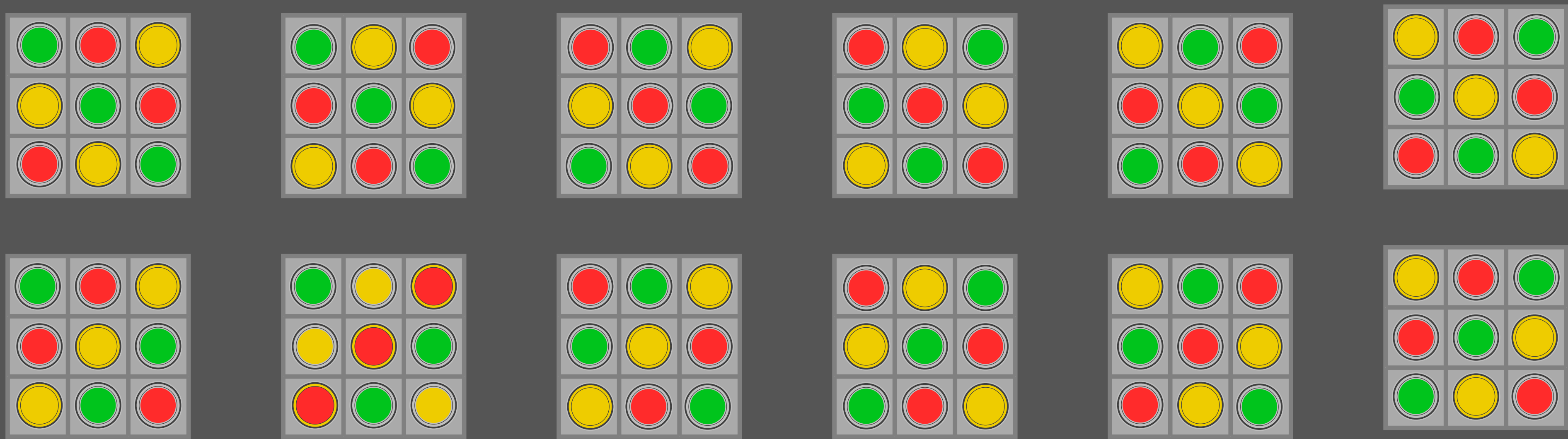
There are really just two different ways of producing a 3 x 3 Latin Square. You can start with your first row and then 'move it along one place' for the next row and then 'move it along another place' for the last row :

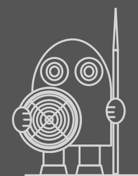


. . . or you can take the same first row and then 'move it along 2 places' for the next row and then 'move it along 2 more places' for the last row :



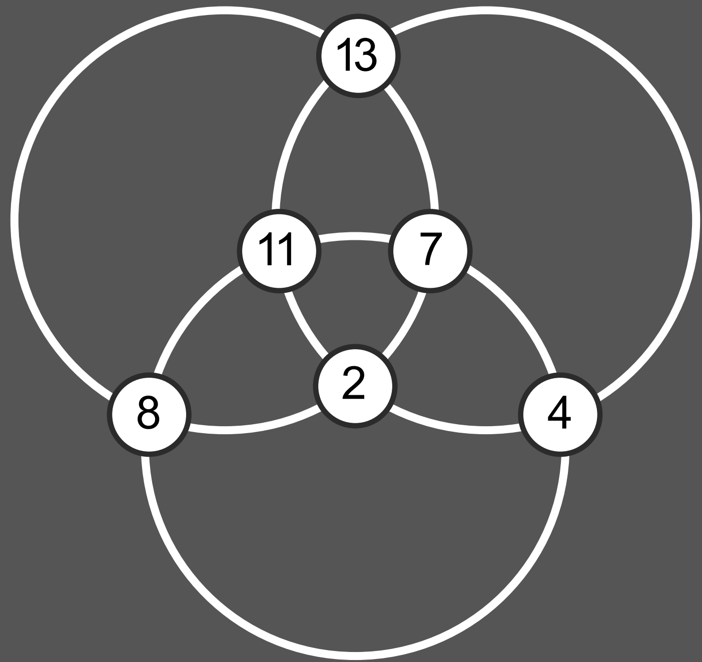
. . . with these two different ways of producing a Latin Square and six different first rows, not surprisingly you will end up with 12 different Latin Squares. How many of these did you find?





ans 5 **Japanese Magic Circles**

There's no obvious way to solve this problem – other than just trying the numbers in different places until you get a feel for how things work. Sooner or later you'll find a successful arrangement. On the right is one answer : if you check it, you'll find that the totals around the three circles are all the same. However . . .

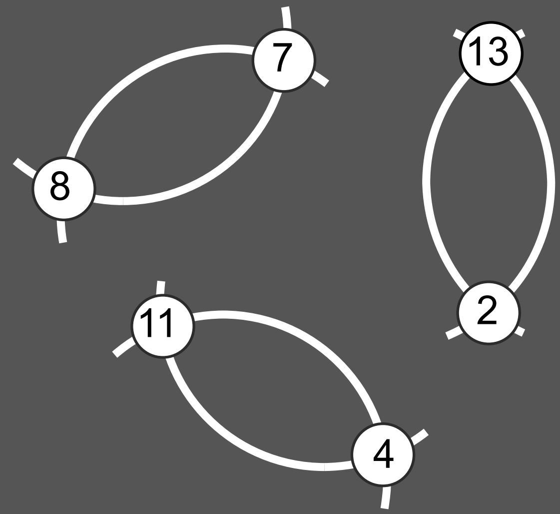


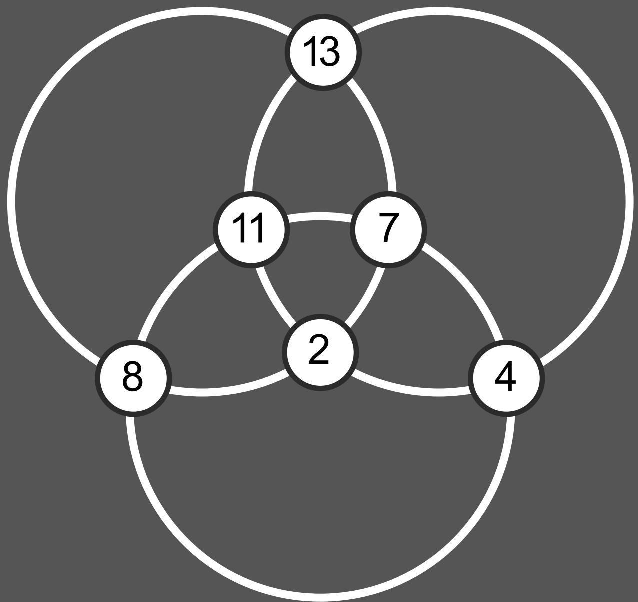
. . . you might have found a different answer – and yes, this is one of those problems where there are a number of different answers, all of them correct.

PTO

ans 5 Japanese Magic Circles

There's an interesting pattern to be found in these Japanese Magic Circles : Starting with the solution given on the previous page, just look at each of the 'rugby ball' shapes in the diagram and add up the numbers at opposite ends, you'll see that you always get the same total, that's to say, 15 !

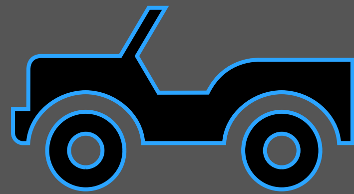




There are many ways of coming to an answer : but it turns out that every correct answer to the problem has these same three pairs of numbers sitting opposite each other. That's to say, if you've got three 'rugby balls', one with 13 opposite 2, one with 11 opposite 4 and one with 8 opposite 7 – then you've got a correct solution !

Perhaps you can see how this simple fact can be used to get you to a correct answer . . .

ans 6 the job's yours !



This is a fairly easy logic problem – as long as you keep a clear head about what's required . . .

There are just three things which the new team-member really has to be : **1** good driver, **2** gun-handler, **3** master of disguise. Everything else we can ignore. Now let's take these three things in turn :

good driver : Pete is ruled out as he can't drive

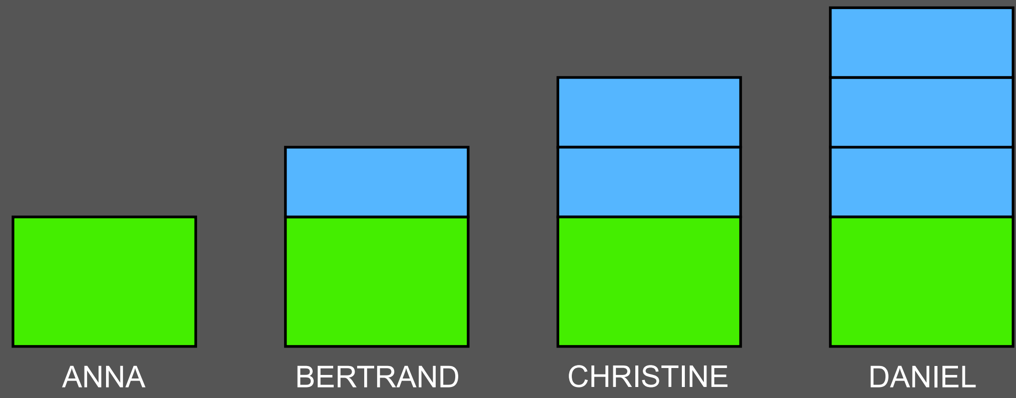
gun-handler : Frankie is ruled out as he can't handle guns

master of disguise : Jake is ruled out as he isn't good at disguise

. . . and that leaves Sam. Which means that Sam will be joining The Major's team.

ans 7 **spaced-out kids**

Let's see if there's a diagram which could help us with this problem . . .



We don't know what Anna's age is, so let's just show it by a simple green box.

We do know that Bertrand is 2 years older than Anna, so we can show his age by the same green box plus something to stand for the extra 2 years he has above Anna : we've used a blue box to stand for 2 years

Christine is 2 years older than Bertrand, so we can show her age as the same as Bertrand's, plus another blue box

And of course Daniel is 2 years older than Christine, so we've given him yet another blue box.

PTO

spaced-out kids

Where do we go from here? Well, we do know that all the ages together add up to 28. Looking at the diagram, this must mean that :

4 green boxes plus 6 blue boxes makes 28

But the blue boxes are each worth 2, so six of them must equal 12 . This means that:

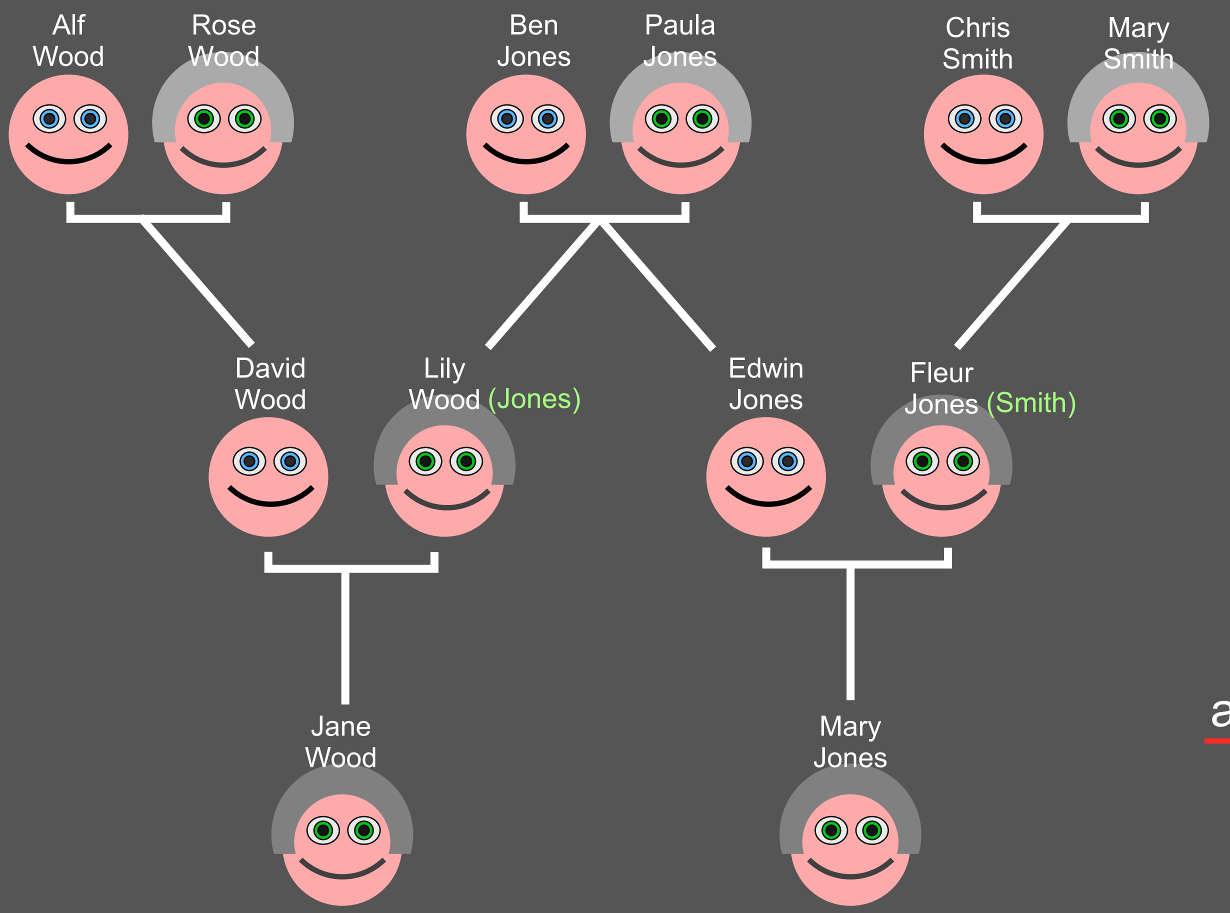
4 green boxes plus 12 equals 28

– which means that the 4 green boxes must add up to 16. In other words, each green box must be worth 4. From this we can easily work out that :

Anna must be 4 and so Bertrand must be 6.

ANS 8 **Jane's people**

If Jane and Mary are cousins, then one of Jane's parents must be brother or sister to one of Mary's parents. So of course, they'll have just two grandparents in common. Here's one way in which it could happen in real life :

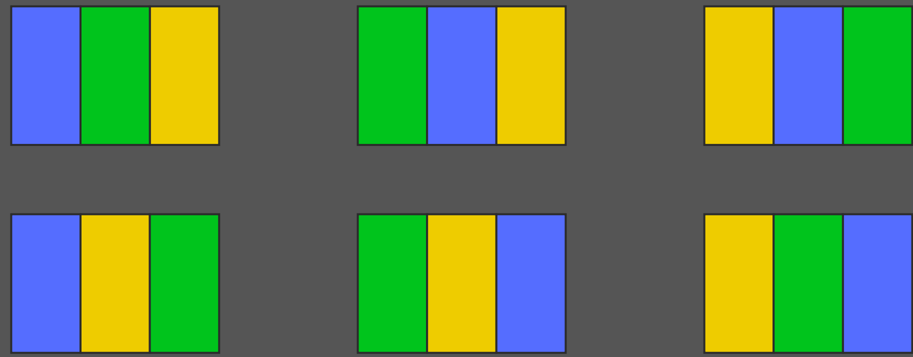


* the names in brackets are the names they were born with

answer : 2

ans 9 **flag day**

- In all, six different arrangements are possible. Here they are :



Without making a list or drawing and colouring a full set of flags, you can get to the same answer by this reasoning :

STRIPE 1		STRIPE 2		STRIPE 3		
3 choices	x	2 choices	x	1 choice	=	6 arrangements

- There are six possible arrangements and, as you can see, just two of them have blue between two other colours. So, we can write :

probability of blue between two others = 2/6 = 1/3

special note : *in maths, we always show probabilities as fractions*

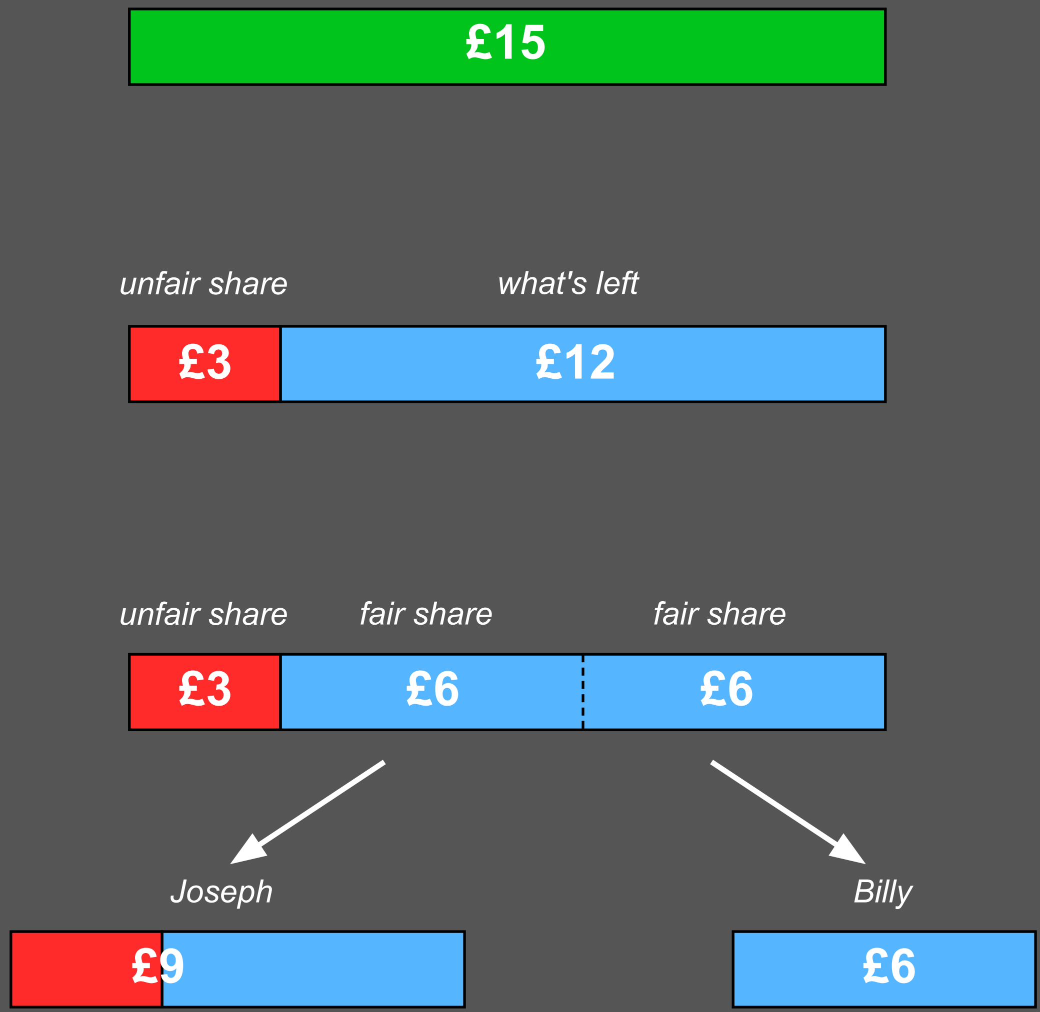
ans 10 it's a clean sweep!

Remember, diagrams are often useful to show what's going on in a problem. Let's start off by drawing a single bar to stand for the £15 which the boys earn :

Now lets split this bar into two parts, a £3 and a £12. The £3 is the 'unfair share' (at least that's what Billy calls it) and this is what Joseph gets. The £12 is what's left after you've removed the 'unfair share'.

If we now take the £12 and divide it equally into two parts (each £6), we can call these parts the 'fair shares'.

Finally, we give the money to the boys: £9 to Joseph (you can see why it's £9) and just £6 to Billy. 'Life is never fair', says Billy.



ans 10 it's a clean sweep!

Some people prefer to set this kind of problem out in a different way – without the need to draw diagrams. So, here's one way of doing that. The most important thing to remember though is that it's usually better to deal with the 'unfair share' first.

Let's keep what the two boys are getting in two separate columns. Also,in maths we like to shorten things, so let's call the fair share the F.sh and the unfair share the Unf.sh . . .

	Billy	Jos.
Unf.sh		3
F.sh		

First we write the £3 Unf.sh under Jos.

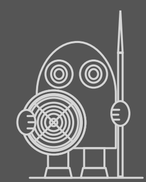
	Billy	Jos.
Unf.sh		3
F.sh	6	6

Next, we divide the £12 we have left over into two equal fair shares of £6 and put these as F.sh

	Billy	Jos.
F.sh		3
Unf.sh	6	6
	6	9

Finally, we add up the columns to get a total for each boy.

Well, not quite finally: we need to check that our answers match what the question is asking!



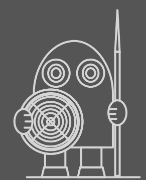
ans 10 it's a clean sweep!

So far we've looked at two different ways of tackling this problem, that's to say using diagrams and setting things out in a table. Of course the numbers in this problem are pretty small, so you might well think that we've made the whole thing unnecessarily complicated. It wouldn't take you long just to try a few numbers until you find an answer which works . . .

. . . for example, suppose we guess that Billy's share is £4. Then we know Joseph gets £3 more than Billy, so Joseph must get £7. But these two amounts, £4 and £7, don't add up to £15; in fact, they add up to just £11. So we need to try again, this time by guessing a larger share for Billy . . .

. . . So, this time let's guess that Billy's share is £5. Then Joseph's share must be £3 more than this, that's to say, £8. Now the two shares add up to £13, so we're getting nearer. Perhaps a guess of £6 for Billy's share will do the trick . . .

Well, this wasn't too hard at all. But you'll find you often come across problems just like this one (we call them 'unfair sharing' problems), except with harder numbers – and then one of the other methods we've shown you might suit you better.



heads and tails

Let's start by taking a careful look at what can happen when you toss two ordinary coins into the air. Perhaps your first reaction is to say that there are only three results possible : two heads, two tails or a head and a tail. That's good as a general way of describing things – but when it comes to calculating probabilities . . .

. . . we need to be very precise. Let's call our two coins SILVER COIN and BRONZE COIN. Now we can see that there are the four equally likely results :

SILVER COIN	H	H	T	T
BRONZE COIN	H	T	H	T

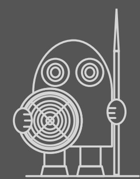
- The first question asks us to work out the probability of getting two heads. We can see straight away that this happens just once out of the four possible results. So, prob (two heads) = 1/4

- The second question asks us to work out the probability of getting a head and a tail. We can see that this happens twice out of the four possible results. So, prob (a head & a tail) = 2/4 = 1/2

ans 12 will there be time ?

Here are all the palindromes the digital clock will show between midnight one night and midnight the following night. Altogether there are 16 different ones . . .

00:00	10:01	20:02
01:10	11:11	21:12
02:20	12:21	22:22
03:30	13:31	23:32
04:40	14:41	
05:50	15:51	



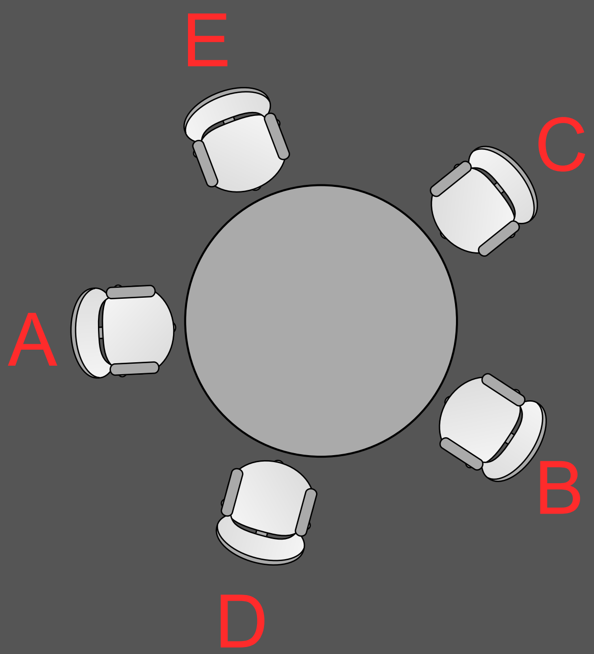
ans 13 mean Mr Francois

First of all, remind yourself of how we get mean averages in the first place : we add up a set of figures and then divide this total by the number of figures we've added. For example, suppose you want to average these four numbers : 3, 7, 10 and 12. Adding these numbers together gives a total of 32. There are 4 numbers here, so we divide our total by 4; and since 32 ÷ 4 = 8, our (mean) average = 8.

In the Mr Francois problem, we can't begin by just adding up the numbers, as we don't have them all. But we do know that there were five numbers – and we do know that their average = 7. So the total must have been 35 (since 35 ÷ 5 = 7). Now we're almost there! The four numbers we are given add up to 25, so the missing number must = 10. Friday must have been a very bad day indeed for Mr Francois (and for his pupils).

answer : on Friday Mr Francois got angry 10 times.

ans 14 come round for a meal !



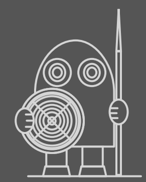
On the left is one way to arrange the seating so that everything works as it should. Your diagram might look a little different from this but if your letters are in the same order as you go round the table, then it's a good answer. For example, starting at A and going clockwise, you should get the order :

A E C B D

Answers to the questions :

1 Charles is sitting on Ellie's left

2 Beatrice is sitting on Diana's right



digit-sums

- *with numbers larger than 400 but smaller than 500, the smallest digit-sum is 5 :*

  coming from either 401 or 410

- *with numbers larger than 400 but smaller than 500, the largest digit-sum is 22 :*

  coming from 499

- *with numbers larger than 400 but smaller than 500, there are nine ways of getting a digit-sum = 12 :*

  408
  417
  426
  435
  444
  453
  462
  471
  480

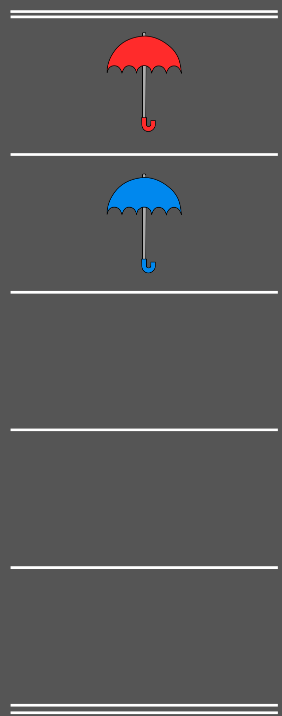
ans 16

singin' in the rain

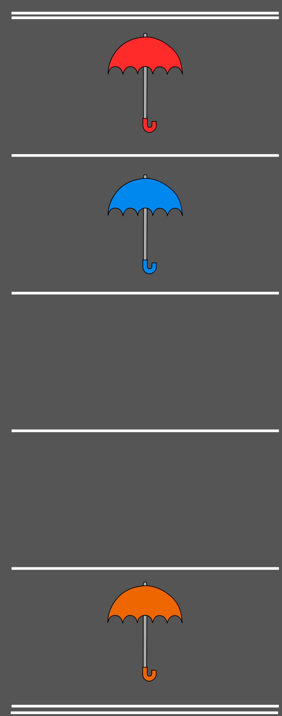
One way to tackle this problem is to make a chart with five spaces (there are six umbrellas but remember, two of them share a place . . . and then to go through what we're given and gradually fill in the chart :

we know that red is first on the list . . .

. . . and we know that blue is second



there were three colours between orange and blue – so orange must be last

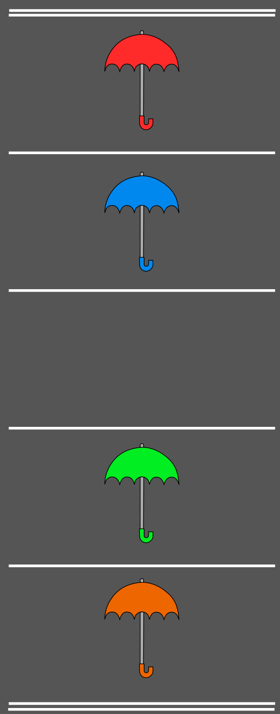


PTO ►►►

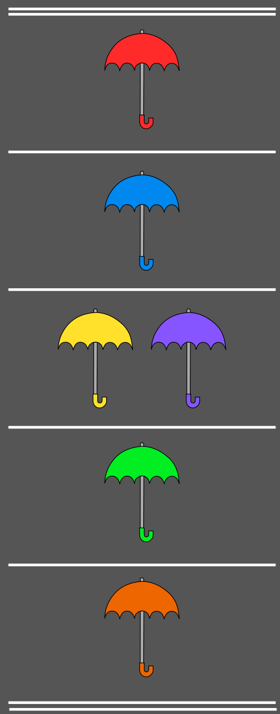
ans 16

singin' in the rain

there were two colours between blue and green – so green must be just above orange



this leaves the last place to yellow and purple (we know they are equal)



– and this final column gives us our answer : red 1st, blue 2nd, yellow and purple 3rd place equal, green 5th and finally, orange 6th ans 17 no red faces here !

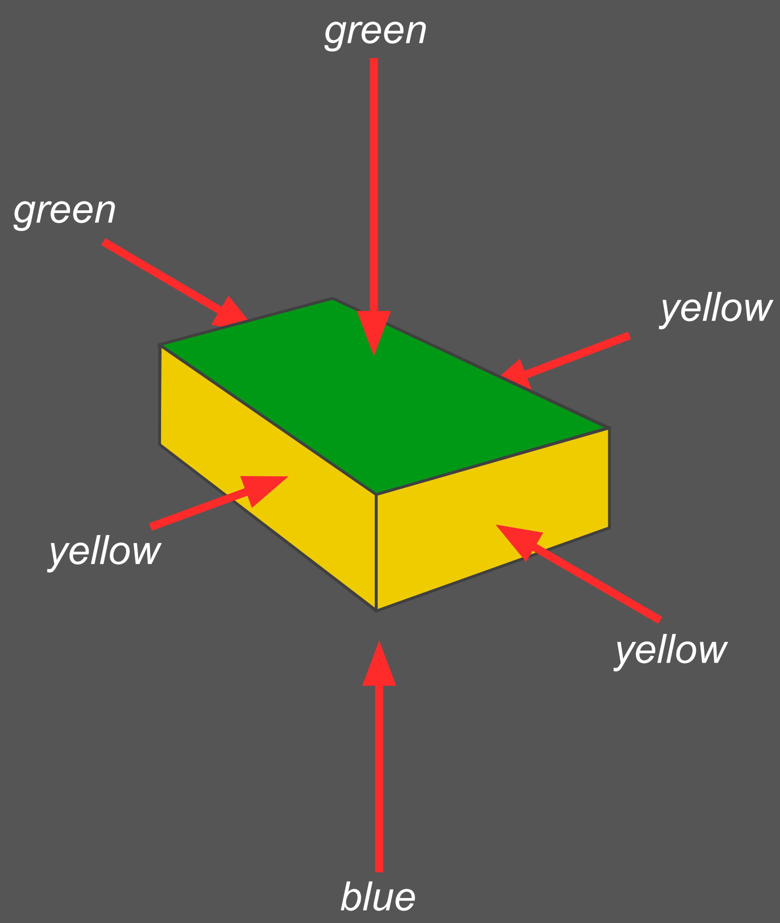
there are two green faces (fact 1), so no long side faces are green and only one end face is green

the top and bottom faces are green and blue

fact 2 is nothing new – it's already covered by fact 1 and what we can see

fact 3 tells us that no long face is blue, so now we can be sure that the end face we can't see and the long face we can't see must both be yellow

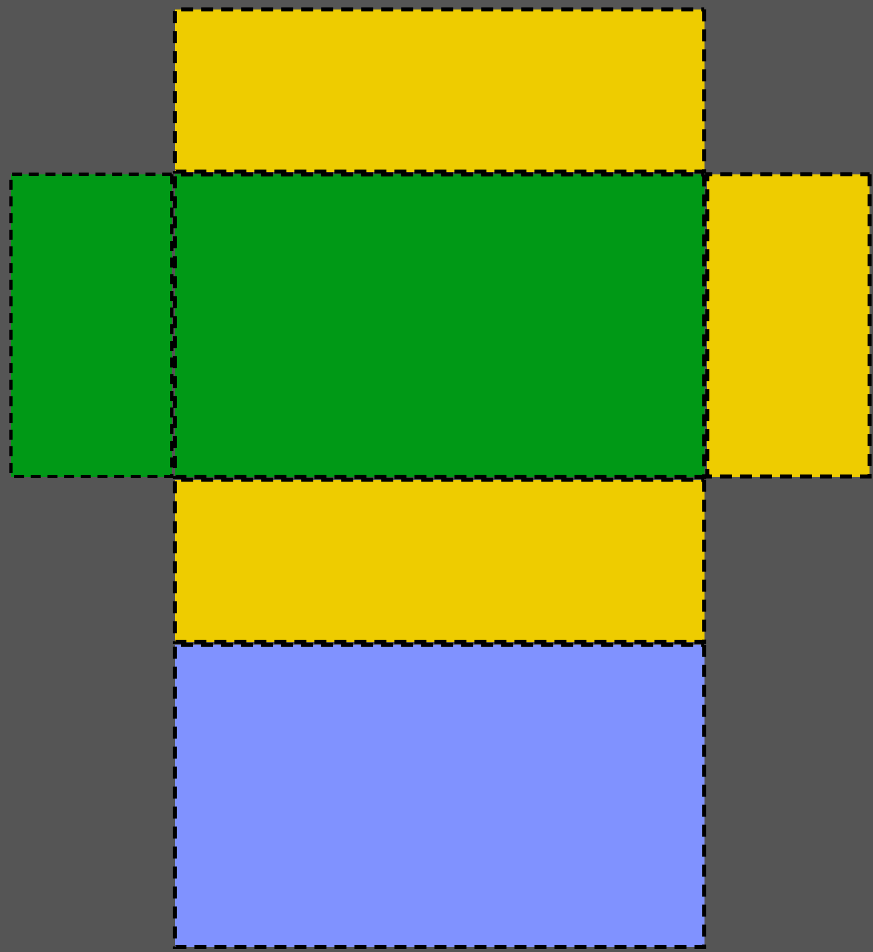
– so altogether there must be three yellow faces (that's two long faces and one end face)



PTO

ans 17 no red faces here !

For anyone who might like a 2D version of the answer, here's a net for making this colourful box :

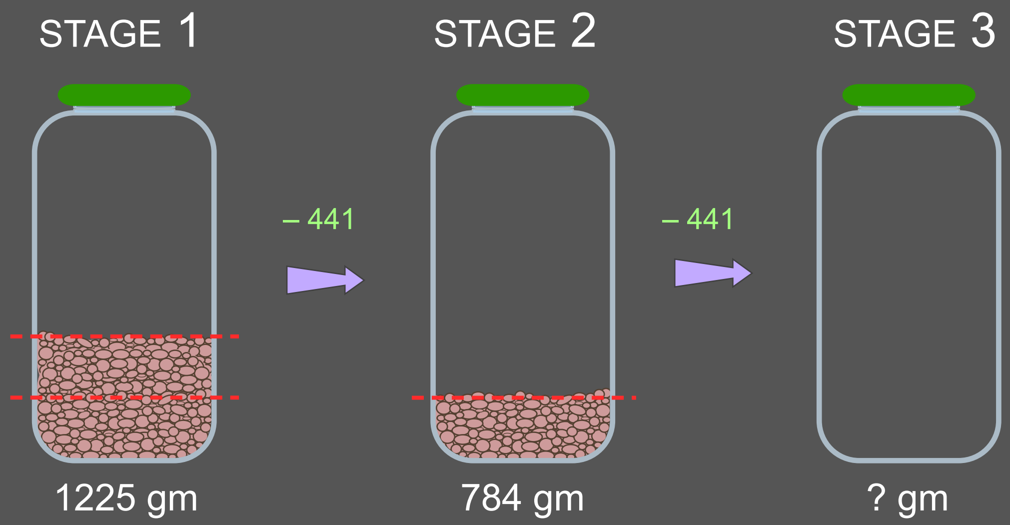


ans 18 completely nuts !

Where to begin? Well, why not draw a diagram?

this makes things much easier !

diagram shows the three stages of the jar and nuts



notice that we've used the same units for all the weights

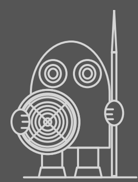
going from stage 1 to stage 2, the jar loses half the nuts

. . . and that equals 441gm (1225 – 784 = 441)

going from stage 2 to stage 3, the jar again loses half the nuts

. . . so once again we lose 441gm

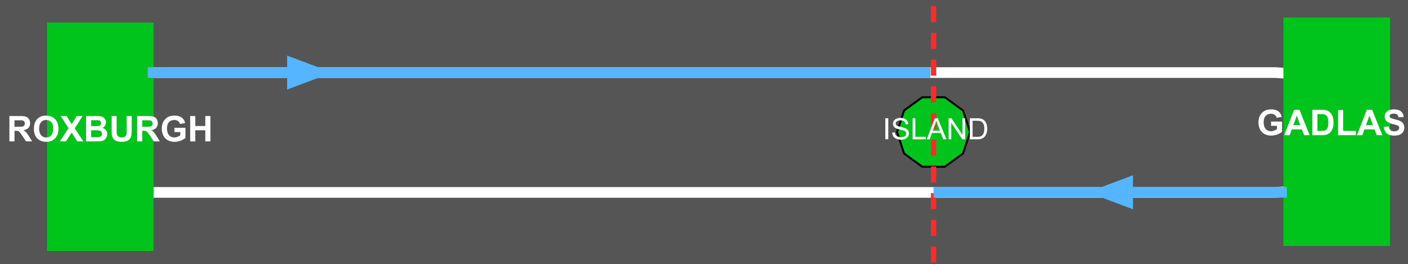
784gm – 441gm = 343gm = weight of empty jar



ans 19 **Annabelle sails**

This diagram shows the two parts of the race

Getting to the island took the same length of time each day, or in other words, the two stretches coloured blue took exactly the same time . . .



. . . but as Annabelle was travelling twice as fast on the first leg, she must have covered twice the distance. So Roxburgh to the island must be twice as far as Gadlas Creek to the island – which means that these two distances (adding up to 18 km, we know) must be 12 km and 6 km.

answer : the small island is 12 kilometres from Roxburgh

ans 20 dockyards & warships

To start with, let's simplify things by calling the four sailors A, B, C and D.

Next, you can anwer this question really easily if you spot the fact that whenever there are three men up on deck, there must be one man down below. In other words, every arrangement of three men on deck must correspond exactly to one particular man below deck. There are just four different ways of having a man below deck, so the answer to the problem is :

Starting with four men, you can make four different groups of three.

The four groups are

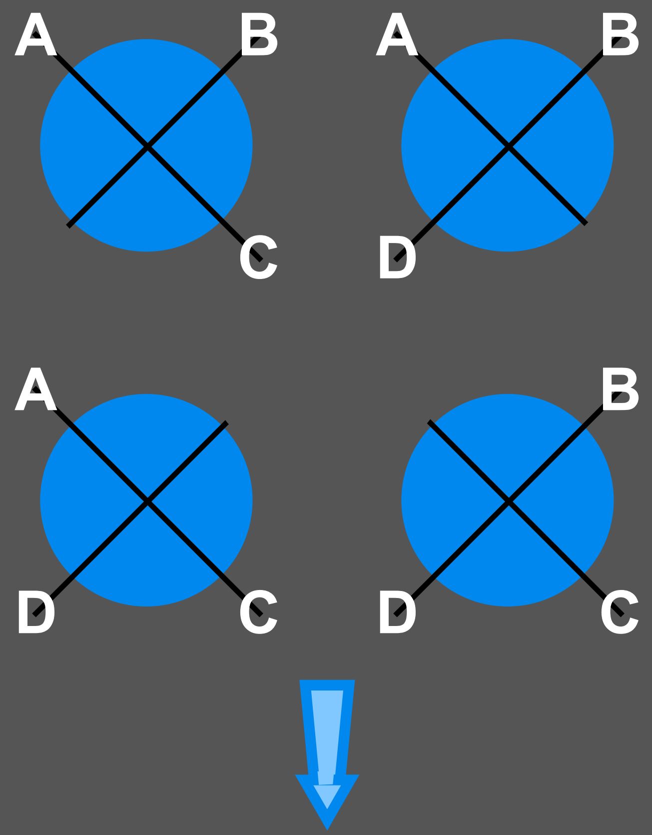
A B C / A B D / A C D / B C D

One way of getting these groups is to make a list with ABCD four times over and then to take out a different letter on each line (that's for the one man below deck). You can see this method illustrated here on the right :

A	B	C	
A	B		D
A		C	D
	B	C	D

PTO ⏩

dockyards & warships



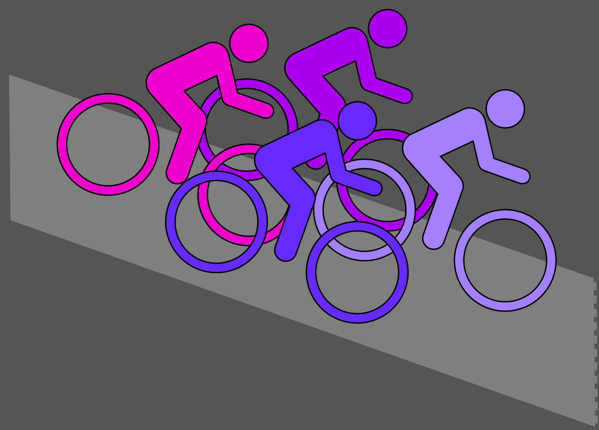
Now here's another way of getting to the answer; this way will appeal to those of you who like to find a diagram approach to problems :

Imagine there's a four-seater table on deck where all four sailors can sit. One by one, you leave out one sailor (the one who must stay below deck) – and you end up with four different arrangements; that's to say, four different groups of three sailors :

A B C / A B D / A C D / B C D

ans 21 **uphill & downhill**

There are different ways of getting an answer here. You can list the downhill sections and record for each one how much descending is involved. Adding these figures together will give you the total amount of going down. Then you can list the ascending sections and write down for each one how much climbing is involved. Again, adding these together will give you a total, this time the total amount of climbing. Subtract one total from the other and you've got your answer!

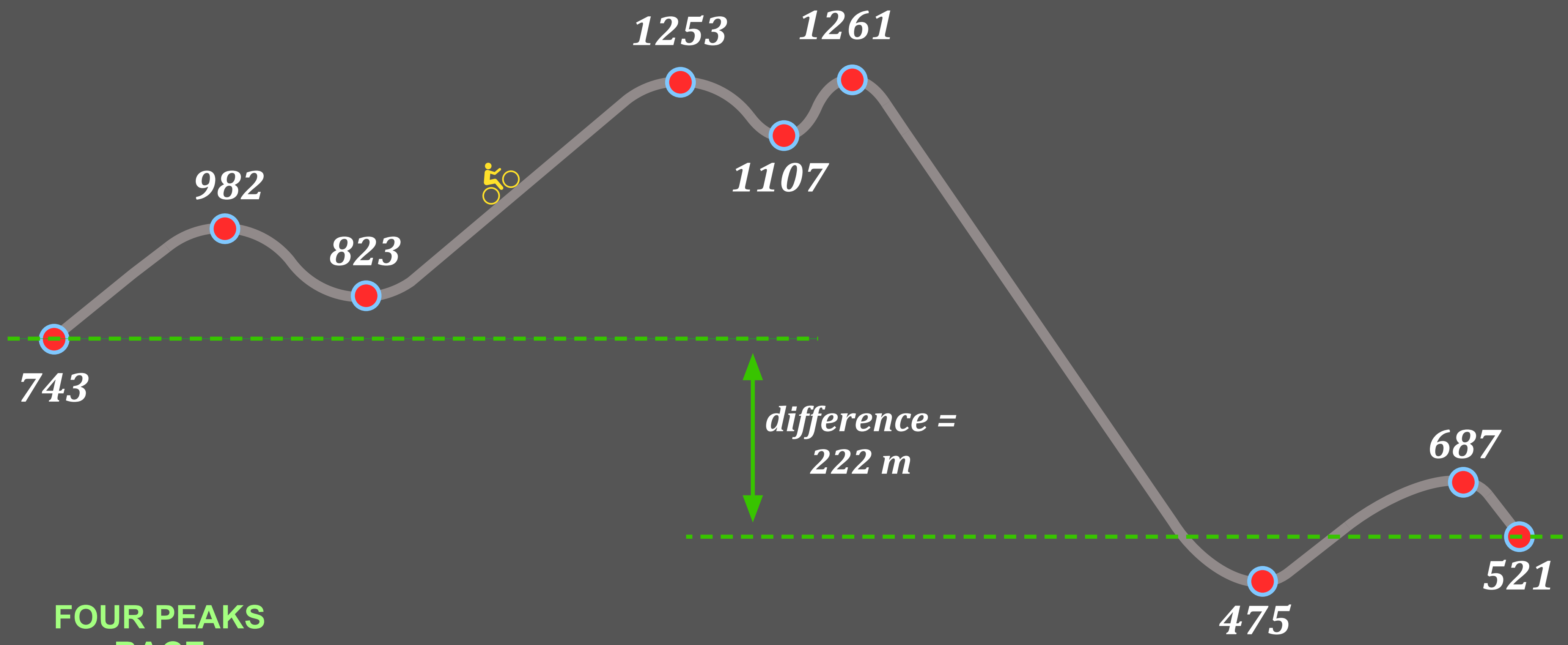


. . . Or, as the diagram (page following) makes clear, you can just take the height of the finishing point and the height of the starting point – and then simply subtract the one from the other . . .

Either way, your answer should be 222 m.

PTO

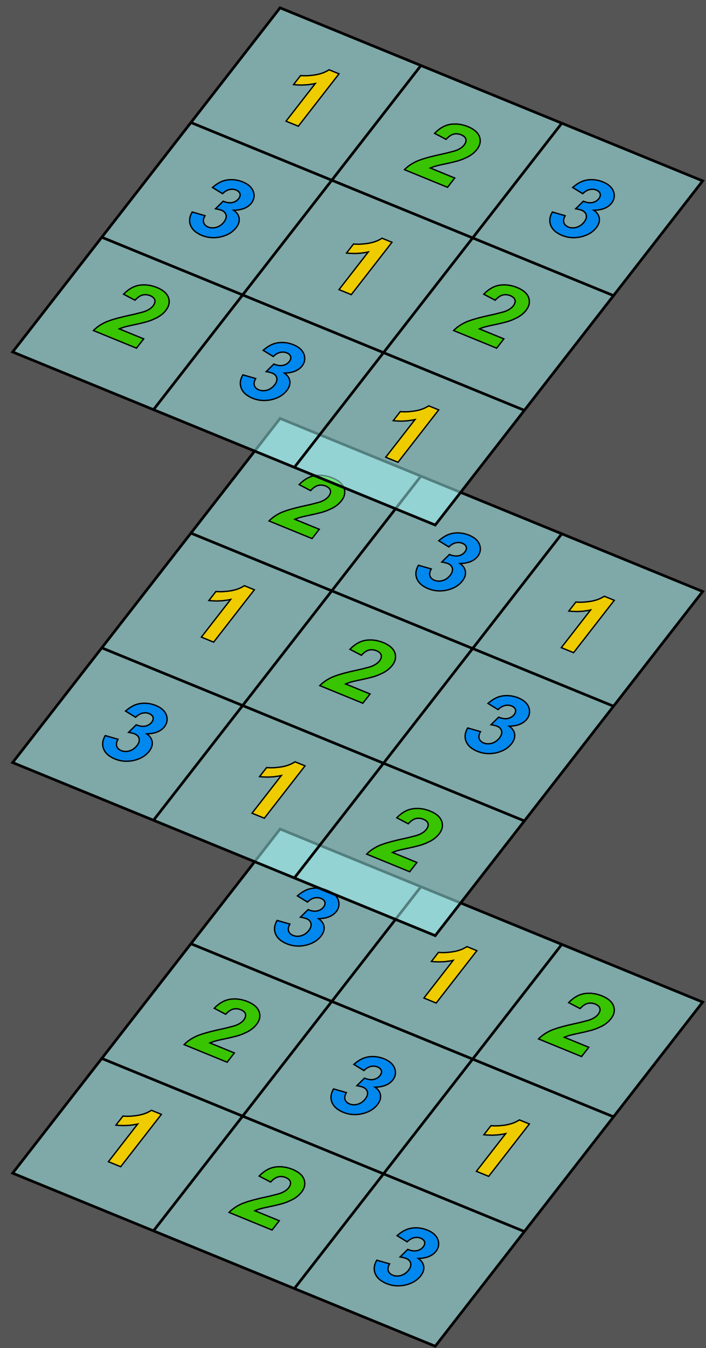
ans 21 uphill & downhill



FOUR PEAKS
RACE

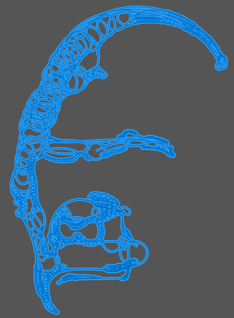
ans 22 latin cube

Here's one arrangement which works :



ans 23 just two gorillas

Perhaps the easiest way of solving this problem is just to pick a number which both 10 and 15 divide into. Let's choose 30, as that's the easiest. So, if we had 30 kg of food left, we could say :



Sultan will eat 30 kg in 10 days = 3 kg per day

Solomon will eat 30 kg in 15 days = 2 kg per day

so, Sultan + Solomon together will eat 3 + 2 = 5 kg per day

And with 30 kg to start with, 5 kg per day would last 6 days . . .

important note *You don't have to choose 30 as your starting number – you could just as well choose 60 or 90 or any other number which both 10 and 15 divide into. Whichever of these numbers you choose, you'll get the same answer . . . try it !*

PTO ▶▶

ans 23 just two gorillas

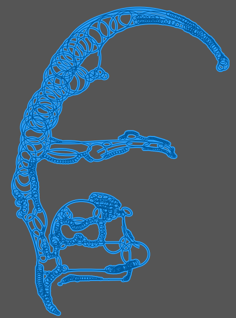
If you're comfortable with adding fractions, you might prefer to solve this problem more directly, like this :

Let's call the total amount of food we've got t kilograms. Then we can say :

Solomon eats each day $= t/10$

Sultan eats each day $= t/15$

so together these two eat $t/10 + t/15$



and $t/10 + t/15 = 3t/30 + 2t/30 = 5t/30 = t/6$

– in plain English, this is to say that the two of them together will eat one-sixth of the remaining food each day . . . which means that this amount of food will last them exactly 6 days.

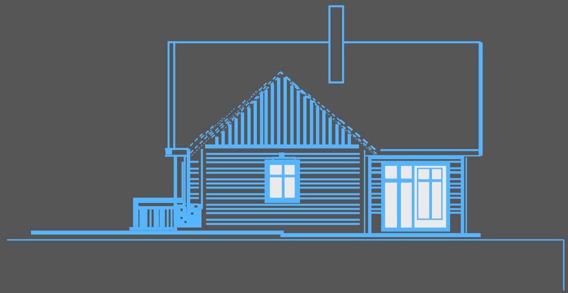
answers :

a. 15 can be made either from a pair of consecutive numbers (7 + 8) or from three consecutive numbers (4 + 5 + 6).

b. 30 can be made either from three consecutive numbers (9 + 10 + 11) or from four consecutive numbers (6 + 7 + 8 + 9).

c. No you certainly can't ! If you add any two consecutive numbers, you'll get an odd number – but if you add four consecutive numbers, you'll always get an even number. No number can be both odd and even ! So there's no answer to this one . . .

the little house on the cliff . . .



20% of 100,000 is 20,000

So at first the value of the house fell by £20,000

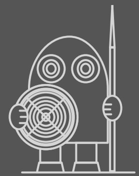
100,000 – 20,000 = 80,000

In other words, after the bad news about unstable cliffs, the value of the house fell to £80,000.

The new geological survey led to an increase in value from £80,000 back to £100,000 – that's to say, an increase in value of £20,000

But 20,000 is 25% of 80,000.

So the answer is : When the value of the house returned to its original level, it went up by exactly 25%.



ans 26 sum of three primes

This question seems to be just about prime numbers but it's also about odds and evens . . . and the important thing to remember here is that when you're adding numbers, (1) *two odds or two evens make an even* and (2) *an odd and an even together make an odd* . . .

So, how does this affect our question? Well, we know that 100 is even, so to get this by adding three smaller numbers together, we must have either three evens or an even and two odds . . .

We know there certainly aren't three even prime numbers. So we must have two odd primes and an even prime. There is of course only one even prime number and that's 2. And 2 also happens to be the smallest prime number. So now we have our answer . . .

If three prime numbers add up to 100, the smallest of them must be 2.

extra question

With 2 as the smallest, what could the other two prime numbers be?

PTO

ans 26 sum of three primes

With 2 as one of the three primes, the other two must obviously add up to 98. So we're looking for a pair of prime numbers which add up to 98.

The numbers we're looking for must be odd. But we also know that their last digits must add up to 8. All numbers ending in 5, apart from 5 itself, will be multiples of 5, so we needn't bother about them.(The only possible pair involving 5 is the pair 5 + 93 and that's no good because 93 is a multiple of 3.) So our only possible pairs must be chosen from :

either

number ending in 1 plus number ending in 7

or

number ending in 7 plus number ending in 1

Look at the list on the right. As you can see, most of them involve a multiple of 3, so they can't be a pair of prime numbers. What we're left with are the two pairs 37 + 61 and 67 + 31 and since all these four numbers are prime, we have our answer :

The other two primes must be either 37 & 61 or 67 & 31

98	
7 + 91	mult of 3
17 + 81	mult of 3
27 + 71	mult of 3
37 + 61	
47 + 51	mult of 3
57 + 41	mult of 3
67 + 31	
77 + 21	mult of 3
87 + 11	mult of 3

PTO

prime numbers

Here are the prime numbers up to 100. If you're aiming to become seriously good at maths, it's a good idea to learn off by heart at least the prime numbers up to 50. That's not really all that hard to do, especially if you start now . . .

2 3 5 7

11 13 17 19

23 29

31 37

41 43 47

53 59

61 67

71 73 79

83 89

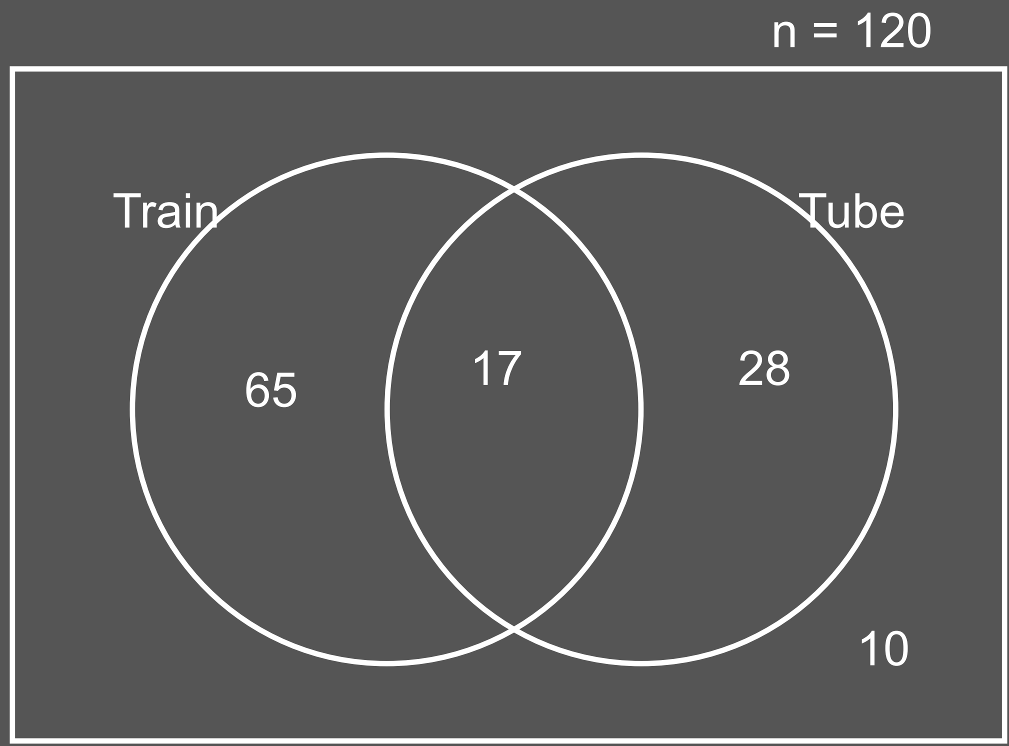
97

ans 27 transport for London

If you add together those who travel by train, those who travel by tube and those who don't travel by either, you get 137. But we know there are only 120 people in the survey.

We have 17 people too many! So, there must be 17 people we've counted twice – or in other words, there must be 17 people who travel by both train and tube. So, there must be 17 people in the intersection set (the set which, as you may know, we can write in symbols as Train ∩ Tube).

* And we know that only 65 people (82 – 17) travel by train alone and only 28 people (45 – 17) travel by tube alone.



$$\begin{array}{r} 82 \\ 45 \\ +\ 10 \\ \hline 137 \\ \hline \end{array}$$

$$\begin{array}{r} 137 \\ -\ 120 \\ \hline 17 \\ \hline \end{array}$$

answer : 17 people travel by both train and tube

ans 28 marking time . . .

You can't average the marks as they stand but if you turn them all to percentages, you'll have the same sort of thing throughout – and then you can work out an average in the usual way. (That's to say, add them all up and divide by 6.)

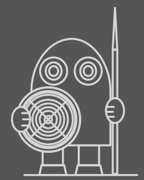
total of all six percentages = 420

so average percentage = 420 ÷ 6 = 70

And that's it : Anthony's average mark was a cool 70%. Not bad !

36 / 60	=	*60%*
48 / 50	=	*96%*
74%	=	*74%*
18 / 20	=	*90%*
30 / 75	=	*40%*
24 / 40	=	*60%*

If you're not sure about turning fractions into percentages, look on the next page . . .



ans 28 marking time . . .

36 / 60 .... cancel : divide top and bottom by 12, which will give you 3/5 ie 60%

48 / 50 .... just double the fraction and you get 96/100 ie 96%

74% .... it's already done for you!

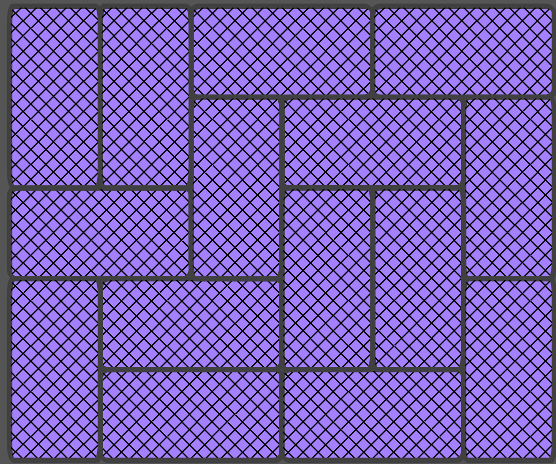
18 / 20 .... multiply top and bottom by 5 and you get 90/100 ie 90%

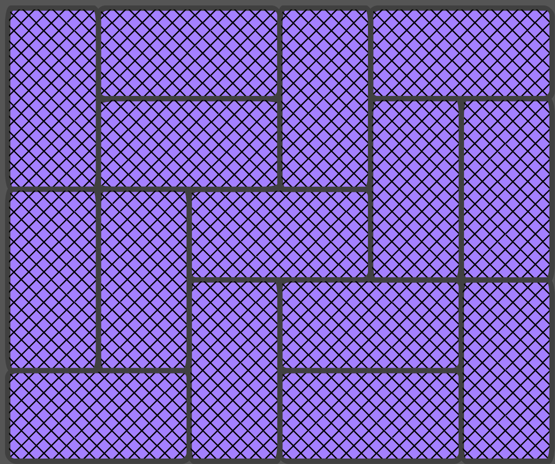
30 / 75 .... cancel : divide top and bottom by 15, which will give you 2/5 ie 40%

24 / 40 .... cancel : divide top and bottom by 8, which will give you 3/5 ie 60%

ans 29 fault lines

There are different ways of solving this problem – here's one way :



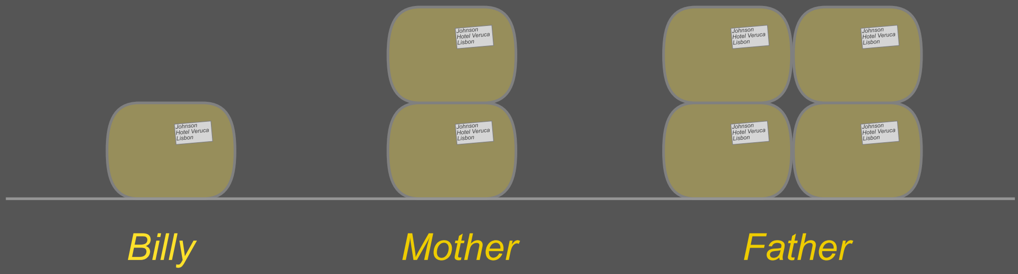


. . . and here's a different way

You might well have found other ways – but do double-check your answers : this is one of those annoying problems where you think you've solved it and then you look again – and see you've still got a fault line!

ans 30 **an open and shut case**

Forget the different shapes of these cases, just think of their weights :



From the information we're given, we know that Billy's luggage weighs half of Mother's luggage – and Mother's luggage weighs half of Father's luggage . . . So, using brown blobs to stand for units of weight, it's like our picture above, which shows : 1 unit for Billy, 2 units for Mother and 4 units for Father.

We're told that the difference between Father's luggage and Billy's luggage is exactly 150 kg – and we can see from the diagram that Father's luggage is 3 units more than Billy's. This means that the units are worth 50 kg each.

So we can say for definite that Billy's luggage weighs 50 kg, Mother's luggage weighs 100 kg and Father's luggage weighs 200 kg.

final answer : Mother's luggage weighs 100 kg

ans 31 what a racket!

To get to the bottom of this question, you really need to understand how to use 'byes' when you don't have the right number of players (eg 8, 16, 32, 64) to organise things simply. Below you can see how it works out when you do have the right number of players. As you can see, it's all pretty straightforward.

64 entrants

round 1 : 32 matches, 32 winners

round 2 : 16 matches, 16 winners

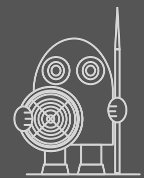
round 3 : 8 matches, 8 winners

round 4 : 4 matches, 4 winners

semi-finals : 2 matches, 2 winners

final : 1 match, 1 winner

On the following page you can see a diagram showing how you could use 'byes' to run a tournament when 11 players take part. Make sure you can see how this works.



PTO

ans 31 what a racket!

11 players enter

ROUND 1

5 players have a 'bye'

6 players play 3 matches (producing 3 winners)

what you're doing here in round 1 produces the same result for the next round (8 players) as if you'd started with 16

ROUND 2

8 players start this round : they play 4 matches (producing 4 winners)

SEMI-FINAL

4 players start this round : they play 2 matches and produce 2 winners

FINAL

2 players start this round : they play 1 match and produce 1 winner

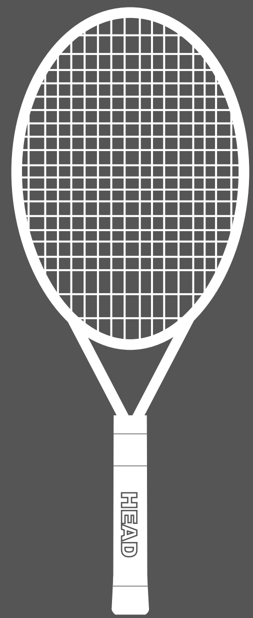
how to use 'byes' in a competition when you're starting with 11 players

PTO

ans 31 what a racket!

By now you perhaps understand how the system of 'byes' works when you're organising a tournament and you don't happen to have eg 8, 16, 32, 64 players. But . . .

. . . you might have spotted something else too. Look back at the last two pages : how many matches *altogether* did it take to finally produce 1 winner? The answer is :



11 entrants – 10 matches in all

64 entrants – 63 matches in all

Yes, that's it! Whatever number of entrants you've got, just subtract 1 and that gives you the number of matches you'll need. Not really a surprise if you stop and think that each match played knocks out exactly 1 entrant.

So, coming back to the original question : Starting with 29 entrants, you'll need 28 matches in total to end up with 1 champion.

ans 32 late for work !

The most obvious way of solving this one is just to think of how many minutes past 7am it was when each of the clerks arrived. Add up all these minutes and divide by eight (because there were eight clerks) and you've got the mean (or average) time past 7am for the group :

$$\begin{array}{r} 20 \\ 35 \\ 26 \\ 22 \\ 36 \\ 27 \\ 37 \\ 29 \\ \hline 232 \\ \hline \end{array}$$

$$8\,\overline{)232}\;=\;29$$

so mean = 29

answer : average arrival time was 07:29

There's another way, which some people find easier – but you need to be happy working with negative numbers. This time you just write down how many minutes before or after 07:30 each clerk arrived and then average these numbers. The advantage of this method is that you're dealing with smaller numbers.

nb Use positive numbers for mins before 07:30 and negative numbers for mins after 07:30.

$$\begin{array}{r} 10 \\ -5 \\ 4 \\ 8 \\ -6 \\ 3 \\ -7 \\ 1 \\ \hline 8 \\ \hline \end{array}$$

$$8\,\overline{)8}\;=\;1$$

so mean = 1 (that's 1 minute early)

answer : average arrival time was 07:29

tickets and pies

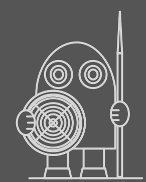
1 It should be easy enough to do this one in your head . . . but in fact many people reading the question and thinking of an answer quickly come up with this answer : 'The programme must have cost £1 and the ticket £10!' It's true that £1 and £10 add up to £11, but this answer won't do! Why not? Because the difference between the two amounts has to be £10 – so this time, the quick answer is a wrong answer! The right answer is of course : programme 50p, ticket £10.50 (These two amounts add up to £11 – and £10.50 is definitely £10 more than 50p, so we we can be sure this is the right answer).

What if you can't easily guess the answer? Is there a way of working it out? Well, you can think of this as an 'unfair sharing' problem – after all, you're just trying to share £11 between a programme and a ticket so that the ticket gets £10 more . . .

UNFAIR SHARING

remember to give out the 'unfair share' first . . .

	programme	ticket
unf. sh.	0	10.0
f. sh	0.50	0.50
total :	0.50	10.50



PTO

2 Another 'unfair sharing' problem! This time you're sharing 47 mince-pies between the family and the School so that the School gets 13 more than the family. The answer is : family 17 pies, School 30 pies. As before, you start by giving the unf.sh (unfair share) of 13 to the School. This leaves 34 mince-pies, so you give a f.sh of 17 to the family and to the School. Then you add up to find the totals. Here's the working-out . . .

	family	school
unf. sh.	0	13
f. sh	17	17
total :	17	30

ans 34 a tricky question

Well, it might not be the smartest way of solving the problem – but one obvious approach is just to try some different numbers for the class size and see where it leads us . . .

FIRST ESTIMATE

Let's suppose the class size is 16. Then we can say there are 8 girls and 8 boys.

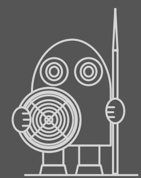
4 girls have their hands up, so 4 must have their hands down

Number of boys with hands down is 1.5 times the number of girls with hands down, so this number of boys must = 6

But we know that all the boys had their hands down, so number of boys in the class must = 6

We've already said that we're assuming 8 girls and 8 boys, so there's a contradiction! Class size = 16 just doesn't work!

What should our next estimate be? You'll probably agree that we need to try a larger number – and of course it has to be an even number. If you think for a moment, you might also see that all our estimates of class size will have to be multiples of 4 (otherwise we'll find ourselves with an odd number for 'girls with hands down' – and we son't be able to do 1.5 times that and get a whole number for the boys total. So our next few estimates should be 20, 24, 28 and so on . . .



PTO ▶▶

ans 34 a tricky question

SECOND ESTIMATE

Let's suppose the class size is 20. Then we can say there are 10 girls and 10 boys.

4 girls have their hands up, so 6 must have their hands down

Number of boys with hands down is 1.5 times the number of girls with hands down, so this number of boys must = 1.5 X 6 = 9

But we know that all the boys had their hands down, so number of boys in the class must = 9

We've already said that we're assuming 10 girls and 10 boys, so again there's a contradiction! Class size = 20 doesn't work! But at least this estimate is better than the first!

THIRD ESTIMATE

Let's suppose the class size is 24. Then we can say there are 12 girls and 12 boys.

4 girls have their hands up, so 8 must have their hands down

Number of boys with hands down is 1.5 times the number of girls with hands down, so this number of boys must = 1.5 x 8 = 12 and so there must be 12 boys altogether in the class

So with this estimate, all the figures work out nicely! And so at last we have our answer :

<u>Altogether, there were 24 pupils in Form 3N.</u>

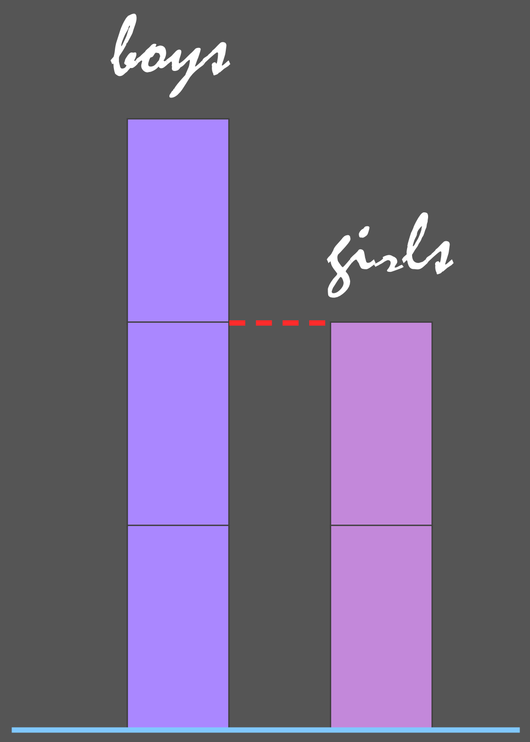
PTO

a tricky question

Of course, some people would prefer a diagram as a good way to see what's happening here . . .

We know that one and a half times as many boys as girls had their hands down. On the right is a simple way of showing this fact :

* We could call this the 'hands down' diagram.



PTO

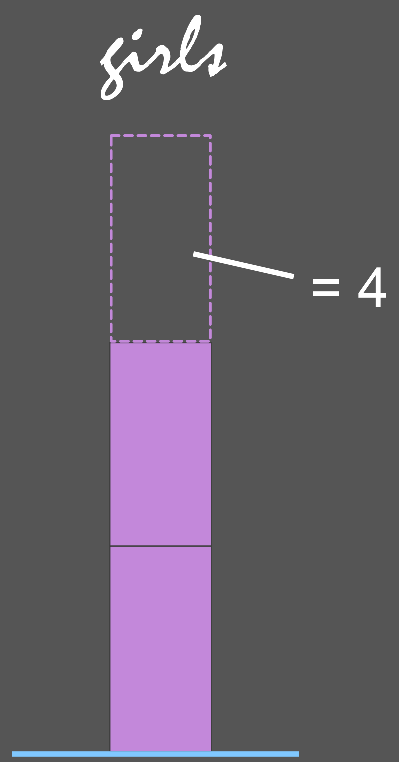
ans 34 a tricky question

In the first diagram we used 3 rectangles to stand for all the boys and just 2 rectangles to stand for all the girls. The rectangle missing on the girls' side stood for those girls whose hands were up.

In the diagram on the right we've shown the 'hands up' girls by an empty rectangle with a dotted line around it. There were 4 girls with their hands up, so of course this rectangle must stand for 4 girls.

This means there must have been 12 girls in all (three rectangles!). There were exactly the same number of boys as girls, so once again there's our answer :

Altogether, there were 24 pupils in Form 3N.



PTO

ans 34 a tricky question

Of course, we don't actually have to have diagrams; we can reason our way to an answer without them. Some of you might well prefer this way of getting from the info we're given to the answer we're after. Here's a purely verbal way of arguing it out :

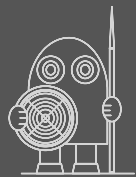
$1\frac{1}{2}$ times as many boys as girls had their hands down

so we could say $3/3$ of the boys had their hands down and $2/3$ of the girls had their hands down

or in other words, $1/3$ of the girls had their hands up

but we know that girls with hands up equals 4 in number; that's $1/3$ of the total number of girls, so this total must equal 12

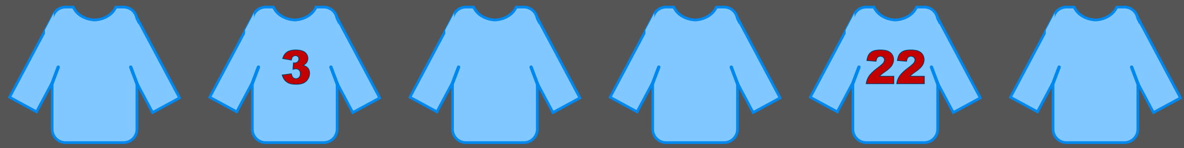
. . . all of which means total class size = 24



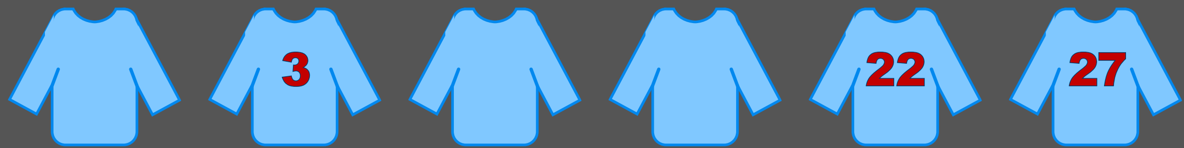
ans 35 6-a-side football

Let's go through the facts we're given and see what we can work out :

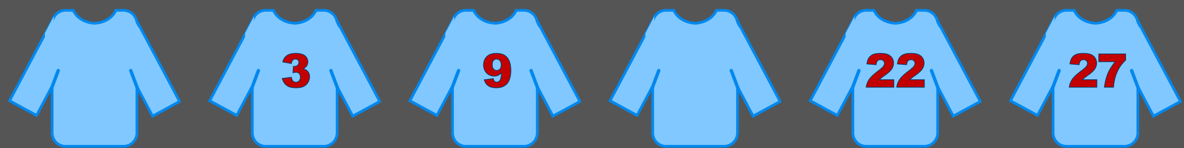
fact 1 tells us that the second number is 3 and the fifth number is 22. So that's a good start :



the fifth number is 5 less than the sixth number (fact 4) : so the sixth number must be 5 more than the fifth, or in other words, it's 27 :



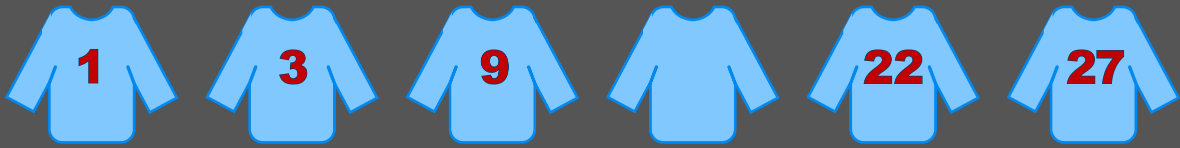
if you square the second number, you'll get the third (fact 3) : that's easy, we all know that $3^2 = 9$



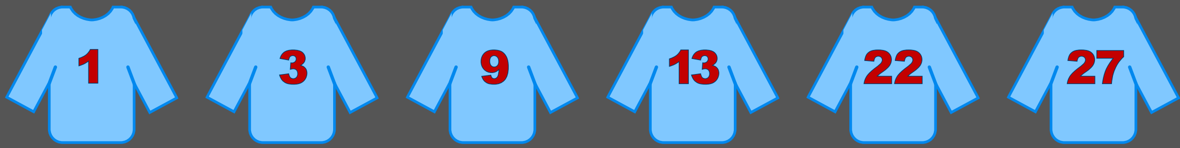
PTO

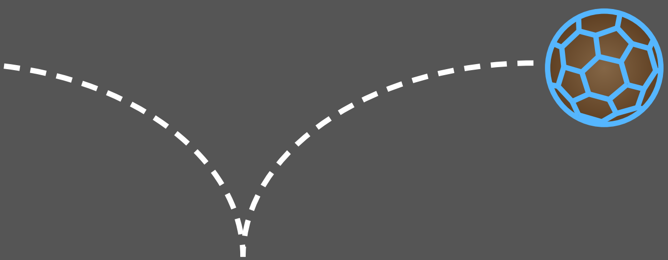
ans 35 6-a-side football

fact 5 tells us that the first three numbers add up to 13 : 3 and the 9 add up to 12, so the missing (first) number must be 1 :



the first four numbers add up to 1 less than the sixth (fact 2). So the first four numbers must total 26, meaning the fourth has to be 13 :





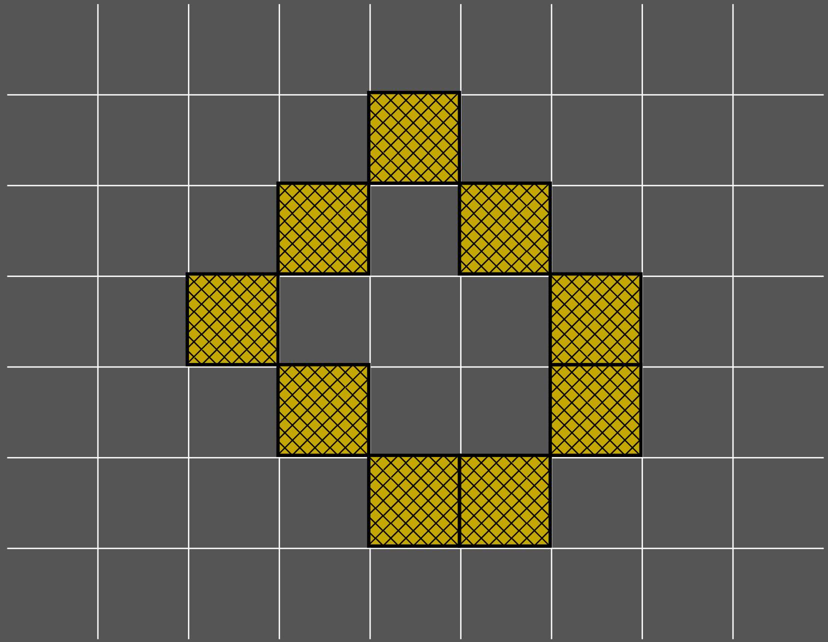
So there we are! The six numbers on the 6-a-side team shirts were 1, 3, 9, 13, 22 and 27.

ans 36 sheep may safely graze

ANSWER : 6 square metres (or 6 squares) seems to be the maximum area you can enclose. Here's one way of doing it :

. . . maybe this is the only way of doing it !

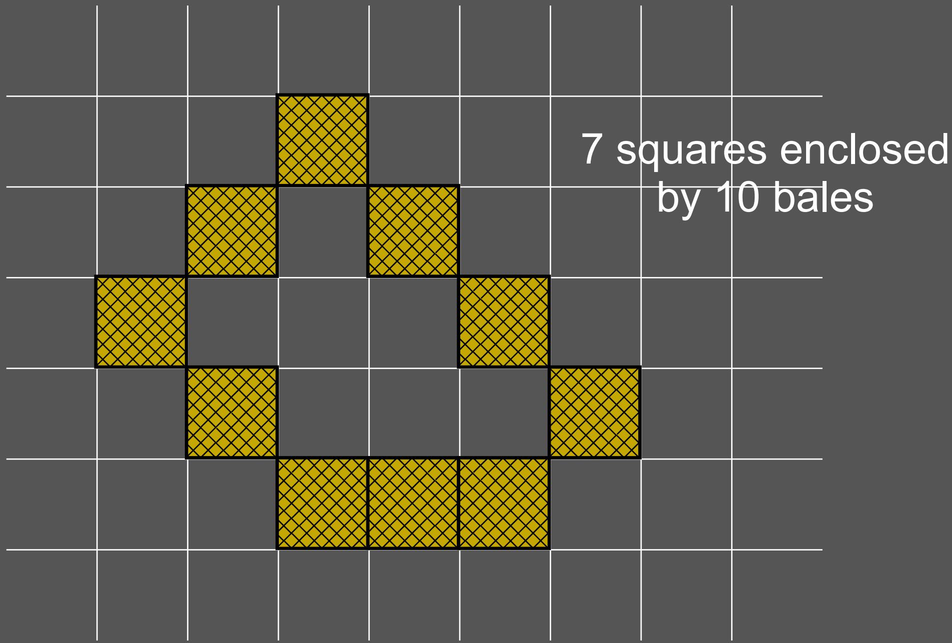
. . . and if you're looking for more sheep pen challenges, try to find a way of enclosing 7 squares using 10 bales – and then perhaps a way of enclosing 8 squares using 10 bales . . .
(answers on the next page)

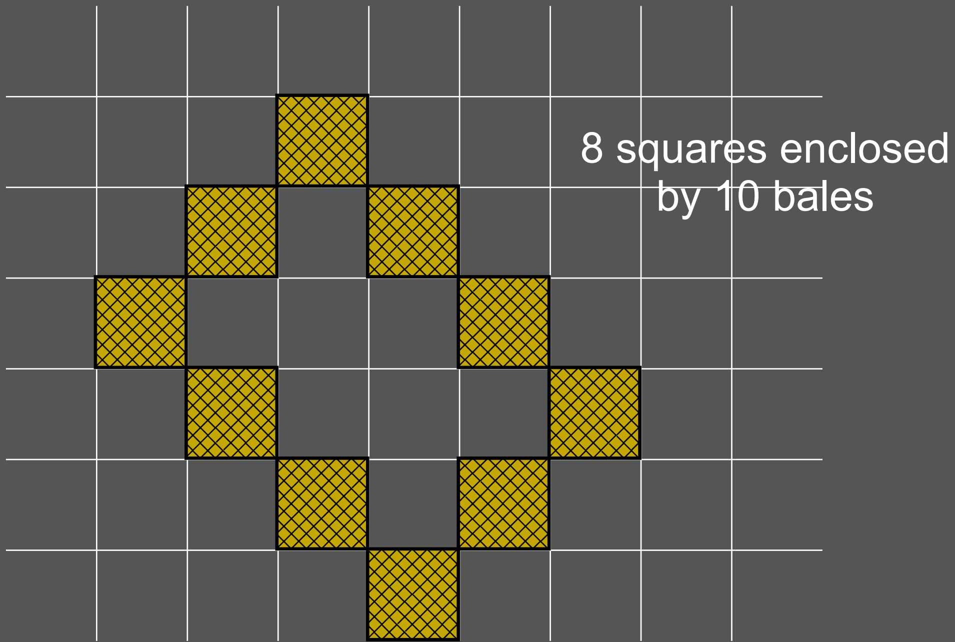


PTO

ans 36 **sheep may safely graze**

Here are possible answers for the two challenges we gave you on the previous page :





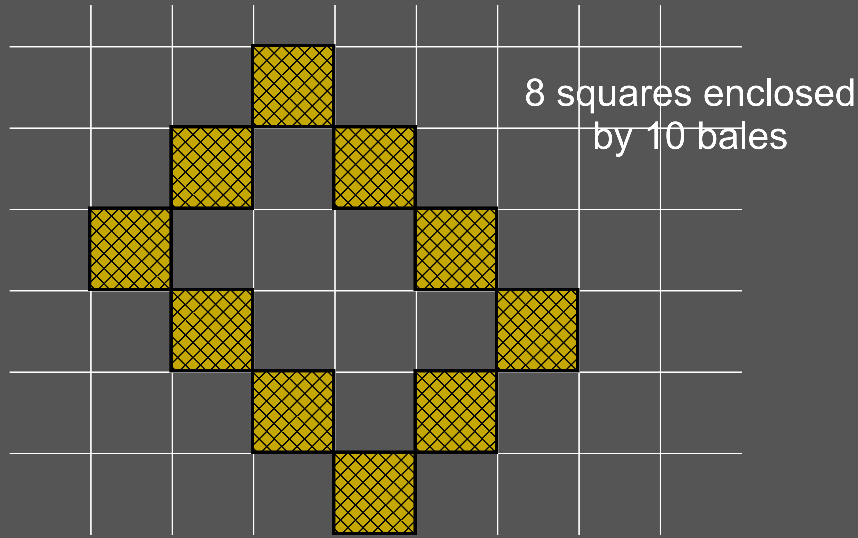
PTO

for two further challenges

ans 36 sheep may safely graze

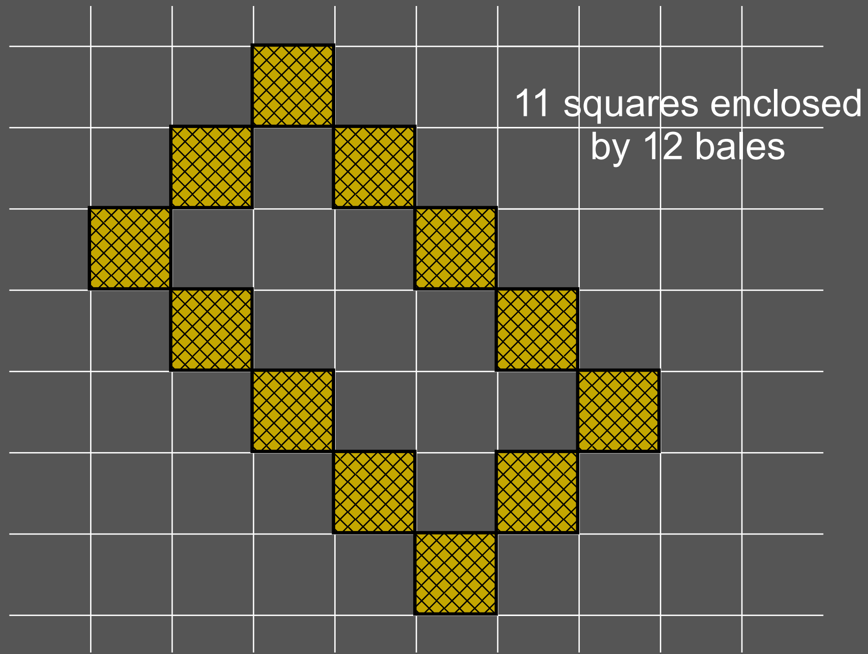
Here we have 10 bales arranged so as to enclose 8 squares. Could you arrange the 10 bales differently so as to enclose 9 squares?

If you think this can't be done, can you suggest a reason why it can't be done?



Now, here's a similar arrangement, this time with 12 bales arranged so as to enclose 11 squares. Do you think there would be any way of arranging the 12 bales differently so as to enclose 12 squares?

Say why you've answered yes or no to the question above.



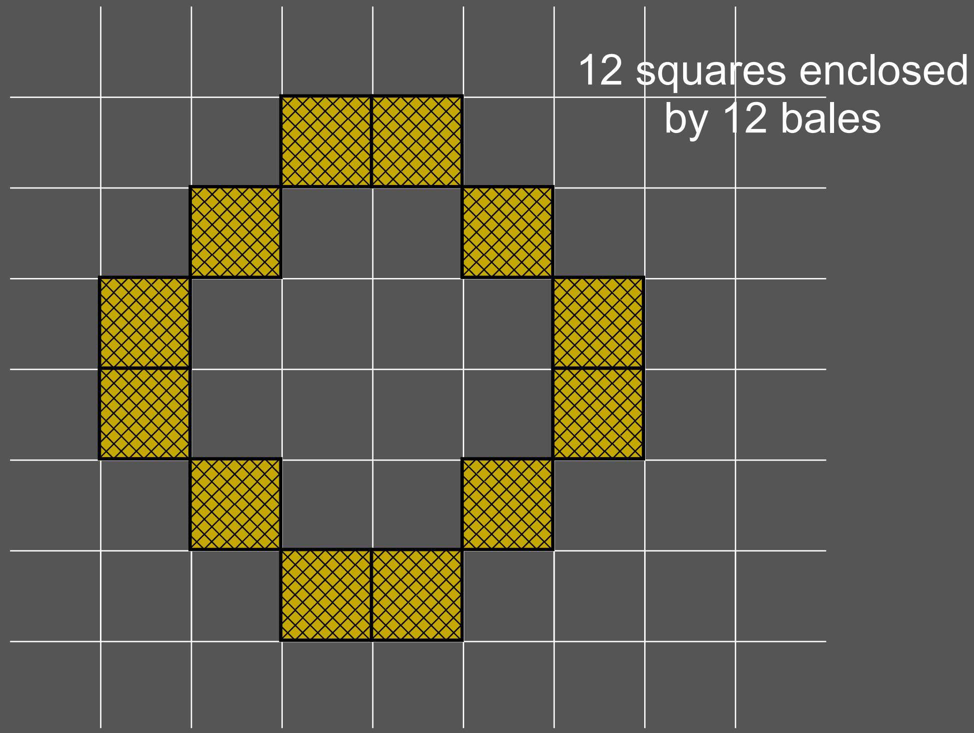
PTO

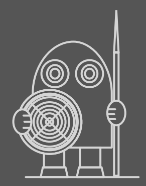
ANS 36 sheep may safely graze

Well, this time it can be done, and here's how :

The difference is that the shape we've enclosed here is nearer to a circle – and any mathematician will tell you that for example if you have a given length of rope, the greatest area you can enclose is by arranging the rope in a circle.

So – just arranging all the bales corner to corner doesn't guarantee that you'll be enclosing the greatest area! You also need to think about getting your bales as close to a circle as you can . . .





ans 37 **Pablo's progress**

To sort this out, we need to be clear what the percentages are percentages of. And that's not too hard once you think about it. For example, the 85% Pablo was given for mental maths just means that he got 85% of the available 20 marks. So we need to calculate 85% of 20 – and that gives us his actual score in the Mental Maths section. This is how it works for the whole maths paper :

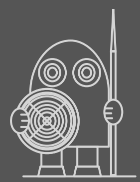
85% of 20 = 17 . . . 17/20

90% of 30 = 27 . . . 27/30

86% of 50 = 43 . . . 43/50

Adding up these three marks gives us a total of 87 out of 100. So that's it :

Pablo's overall maths percentage = 87%



ans 38 happy birthday James !

There are different ways of going about this problem; one easy way is to try some different possible answers until we find what we're after but . . . before we start trying lots of numbers, let's stop and think : If next year, Sophie's age is three times James' age, then Sophie's age must be a multiple of 3. So let's try some multiples of 3 for Sophie and next to them we'll put the corresponding ages for James and after that, their ages last year.

	Sophie	James
next year →	3	1
last year →	1	–

	Sophie	James
next year →	6	2
last year →	4	–

	Sophie	James
next year →	9	3
last year →	7	1

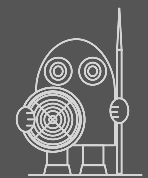
	Sophie	James
next year →	12	4
last year →	10	2

	Sophie	James
next year →	15	5
last year →	13	3

	Sophie	James
next year →	18	6
last year →	16	4

This took a time : we had to wait until our sixth attempt to find numbers which worked out. So finally, if Sophie is 18 next year and James is 6 next year, it all works. Success!

So, that's our answer : next year James will be 6.



ans 39 new year neighbours

How to get started with this problem? You could just keep thinking of numbers which might work and then each time look at the next numbers above and below . . . but that could turn out to be a very slow process. We need a definite way to go about things.

One idea is to start off with numbers in one of the sets involved and look at their neighbours. The square numbers are a good starting point – we know that there are only seven of them below 50 :

1, 4, 9, 16, 25, 36, 49.

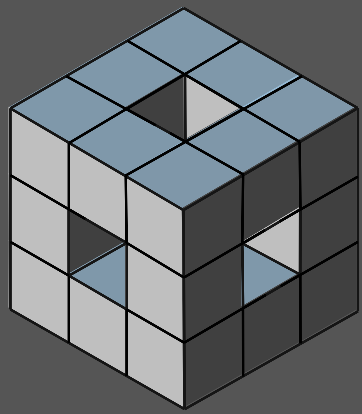
It doesn't take long to spot where the 'special' numbers lie. On the right you can see the list we've made.

answer : 3, 8, 10, 24, 48, 50

PRIME		SQUARE		
		1		
2	3	4		
7	8	9		
		9	10	11
		16		
23	24	25		
		36		
47	48	49		
		49	50	51

ans 40 the heartless cube

a. There are 20 1cm cubes in the shape. You can get this answer either by looking at the diagram and counting carefully – or you can just say there are 27 small cubes altogether in a complete 3 x 3 x 3 cube and this one has 7 cubes missing : that's 1 cube missing from each face (6 cubes in all) and 1 cube missing from the centre, leaving us with 20 cubes.



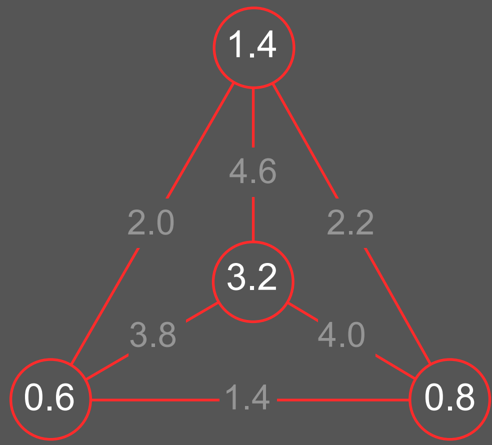
b. The total surface area of the shape is 72 cm^2 (six faces, each with 8 squares showing on the outer surface and six 'holes', each surrounded by four 1cm squares).

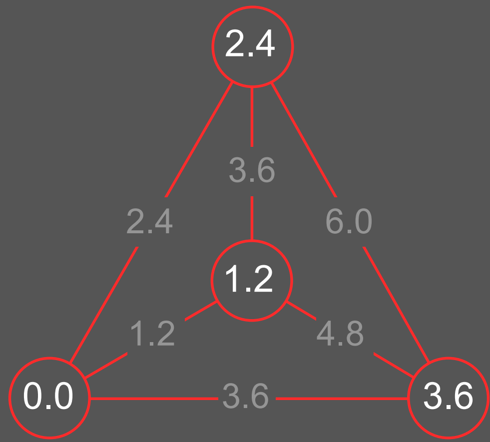
ans 41 more number triangles

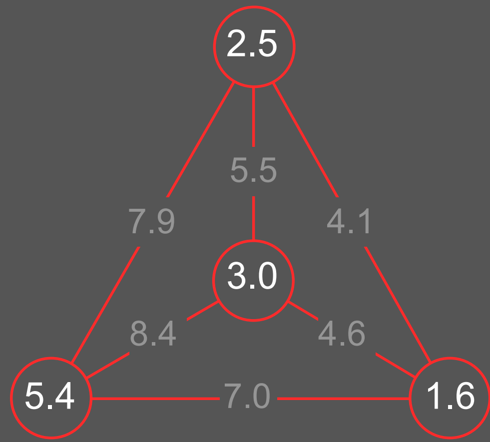
Here are the answers to these three difficult number triangles : the hard way to solve them is just to keep trying numbers until you have some which work – but, as you may have discovered in the easier number triangle questions, a simpler way is to choose one triangle within the shape and then solve that triangle.

* On the next page there are two examples, an easy number triangle and then a harder one, with which we show you how to get started on filling in the blanks . . .

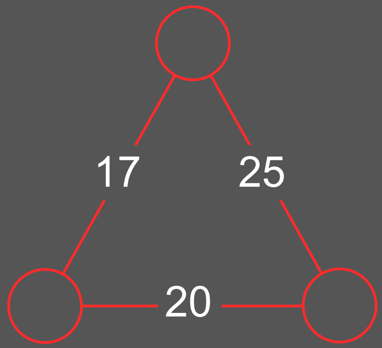
note : some people find it easier here to turn decimals like these into whole numbers – and then to turn them back to decimals at the end.



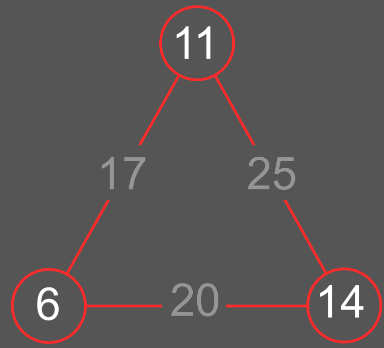


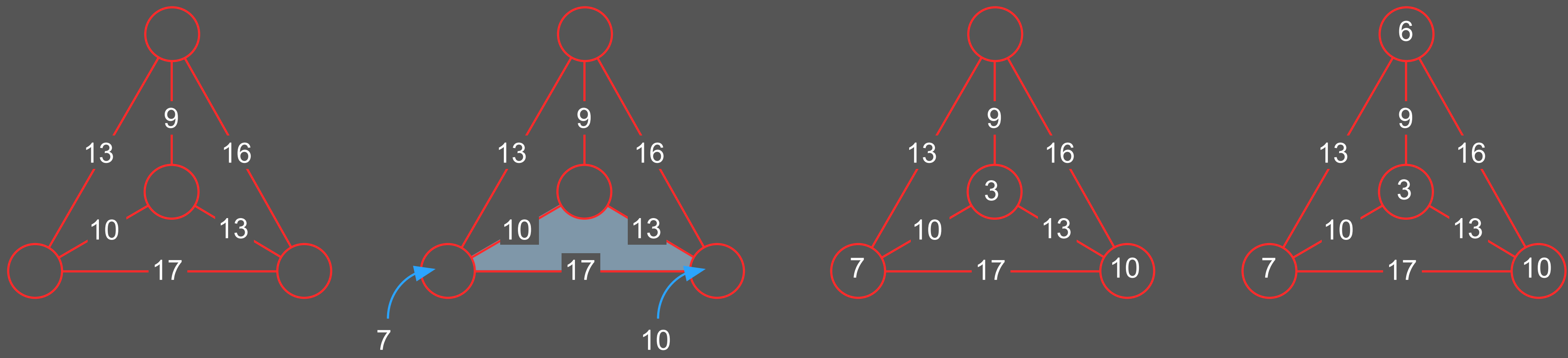


ans 41 more number triangles



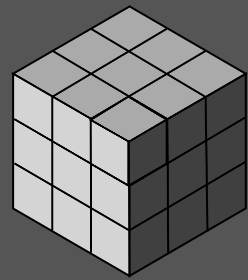
Look at the bottom row : we need two numbers which add up to 20. We can see that 25 is 8 more than 17, so we're looking for two numbers where one is 8 more than the other – can you see why? It's easy then to see we need 6 and 14 :





Now let's use the same method for the this difficult one . . . Let's choose the bottom triangle, the one we've shaded blue. For the bottom circles, we need two numbers which add up to 17 – and which are just 3 apart from each other. 7 and 10 fit the bill (get them the right way round!), so we put them in . . . and immediately we can put 3 in the top circle to complete the bottom triangle. Now we have enough information showing to put 6 on top and complete the whole thing.

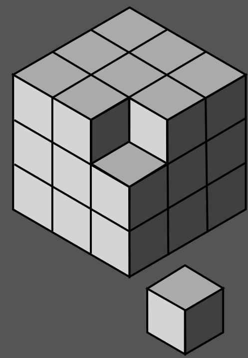
ans 42 the missing cube



- Each face of the cube has an area of 3 x 3 = $9cm^2$

  And the large cube has 6 of these identical faces

  So, surface area of the large cube = 6 x 9 = $54cm^2$



- One way of going about problem 2 is as follows :

  Large cube with one missing corner cube =

  3 faces each with area 9cm2 = $27cm^2$

  3 faces each with area 8cm2 = $24cm^2$

  3 small faces each 1cm2 = $3cm^2$

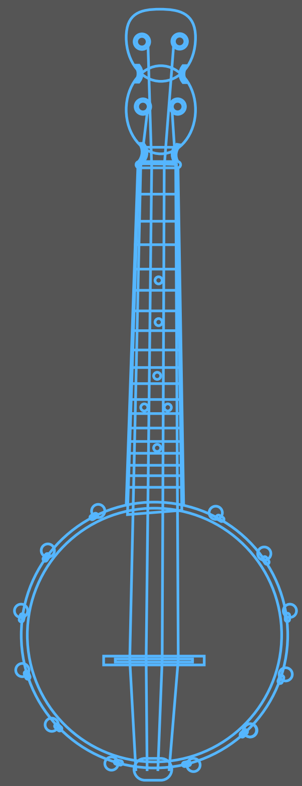
  $54cm^2$

- Perhaps a neater way of solving problem 2 is this : Notice that when you remove one corner cube, you take away three $1cm^2$ outer faces – but at the same time you expose three $1cm^2$ faces which were hidden before. So final result = same as before = $54cm^2$

ans 43 the frog and banjo

There are different ways of going about this problem but here's one way you might like. Begin by making a table like this :

	Frog	*Newt 1*	*Newt 2*	*Toad*
banjo				
lead guitar				
rhythm guitar				
double-bass				



. . . and then work through the information you're given, putting red circles to show where something isn't possible and green circles to show where something must definitely be so . . .

PTO

ans 43 the frog and banjo

First of all, Newt 1 did not play double-bass. So, put in a red circle to show this.

Next, we know that Toad didn't play either lead guitar or rhythm guitar, so put in red circles to show this.

And we do know that Frog played the banjo, so put in a green circle to show this.

Next, we're told that Newt 1 didn't play rhythm guitar. So put in a red circle to show this.

Finally, we know that Newt 2 didn't play lead guitar, so put in a red circle to show this.

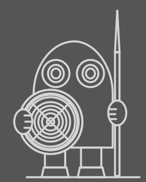
	Frog	*Newt 1*	*Newt 2*	*Toad*
banjo	🟢			
lead guitar			🔴	🔴
rhythm guitar		🔴		🔴
double-bass		🔴		

That takes care of recording the information we're given. The next step is to do some thinking :

1 If Frog played the banjo, he couldn't have played any of the other instruments, could he? So add three red circles to show this.

2 And of course, if the banjo was played by Frog, then it obviously wasn't played by any of the other three band members – so put in three red circles to show this

	Frog	*Newt 1*	*Newt 2*	*Toad*
banjo	🟢	🔴	🔴	🔴
lead guitar	🔴		🔴	🔴
rhythm guitar	🔴	🔴		🔴
double-bass	🔴	🔴		



PTO

ans 43 the frog and banjo

And we're almost there! We know what Frog played but if we look down the column for Newt 1, we can see three red circles and one blank space (level with lead guitar). Newt 1 must have played lead guitar; so put in a green circle to show this.

If we look further along, we can see the same thing applies to Toad : there are three red circles in his column and one blank space. The blank space is level with double-bass, so Toad must have played double-bass. Put in a green circle to show this.

	Frog	Newt 1	Newt 2	Toad
banjo	🟢	🔴	🔴	🔴
lead guitar	🔴	🟢	🔴	🔴
rhythm guitar	🔴	🔴		🔴
double-bass	🔴	🔴		🟢

And finally . . . there's only one player (that's Newt 2) without an instrument and there's only one instrument (rhythm guitar) without a player. So obviously Newt 2 must have played rhythm guitar . . .

. . . and our table is complete !

	Frog	Newt 1	Newt 2	Toad
banjo	🟢	🔴	🔴	🔴
lead guitar	🔴	🟢	🔴	🔴
rhythm guitar	🔴	🔴	🟢	🔴
double-bass	🔴	🔴	🔴	🟢

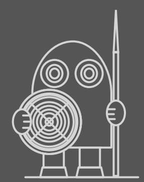
FINAL ANSWER :

Frog played banjo

Newt 1 played lead guitar

Newt 2 played rhythm guitar

Toad played double-bass



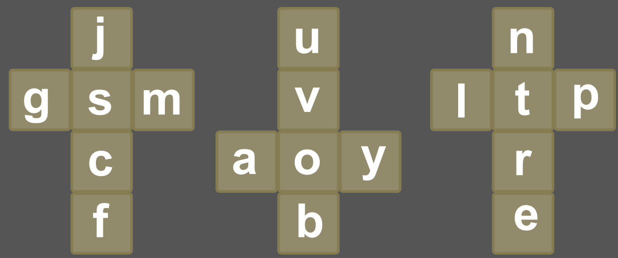
ans 44 **cube calendar - months**

we found an answer – but how did we get it?

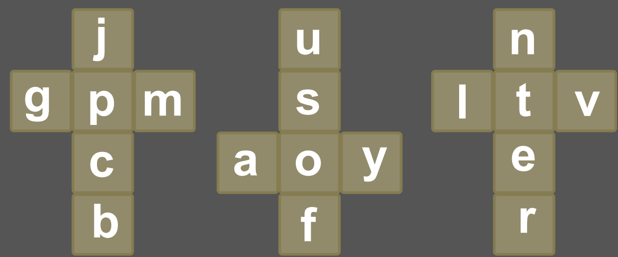
a b c d e
f g h i j
k l m n o
p q r s t
u v w x y z

19 letters needed – so perhaps not too hard to fit onto 3 cubes (18 faces) – especially as some can double up ('d'and 'p' or 'u' and 'n')

we started off with a trial-and-error approach :



<<—— this one can't do 'april'



<<—— this one can't do 'may'

. . . but soon decided a more systematic approach was needed . . .

PTO ▶▶

ans 44 cube calendar - months

First of all we made a vertical list of all 12 months of the year, using just 3-letter abreviations : jan, feb, mar . . . and so on.

j	a	n
f	e	b
m	a	r
a	p	r
m	a	y
j	u	n
j	u	l
a	u	g
s	e	p
o	c	t
n	o	v
d	e	c

Then we looked at our list and started swapping letters around to try to get every letter a in one column, every letter j in another column, every letter p (= d) in another column . . . and so on. Here's the vertical list we ended up with :

a	j	n
f	e	b
a	r	m
a	r	p
a	y	m
u	j	n
u	j	l
a	g	u
s	e	p
c	o	t
v	o	n
c	e	d

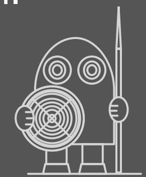
Now we knew that letter v had to go onto the first cube, letter j had to go onto the second cube and letter l had to go onto the third cube. Next, we knew that a, e and p had to go onto the first , second and third cubes respectively. And in this way we continued . . .

v	j	l
a	e	p
f	r	b
u	y	n
s	g	m
c	o	t

So, here's a plan of the three cubes, showing where we put the letters on each. (Notice that u and n had to be on separate cubes, just because of jun)

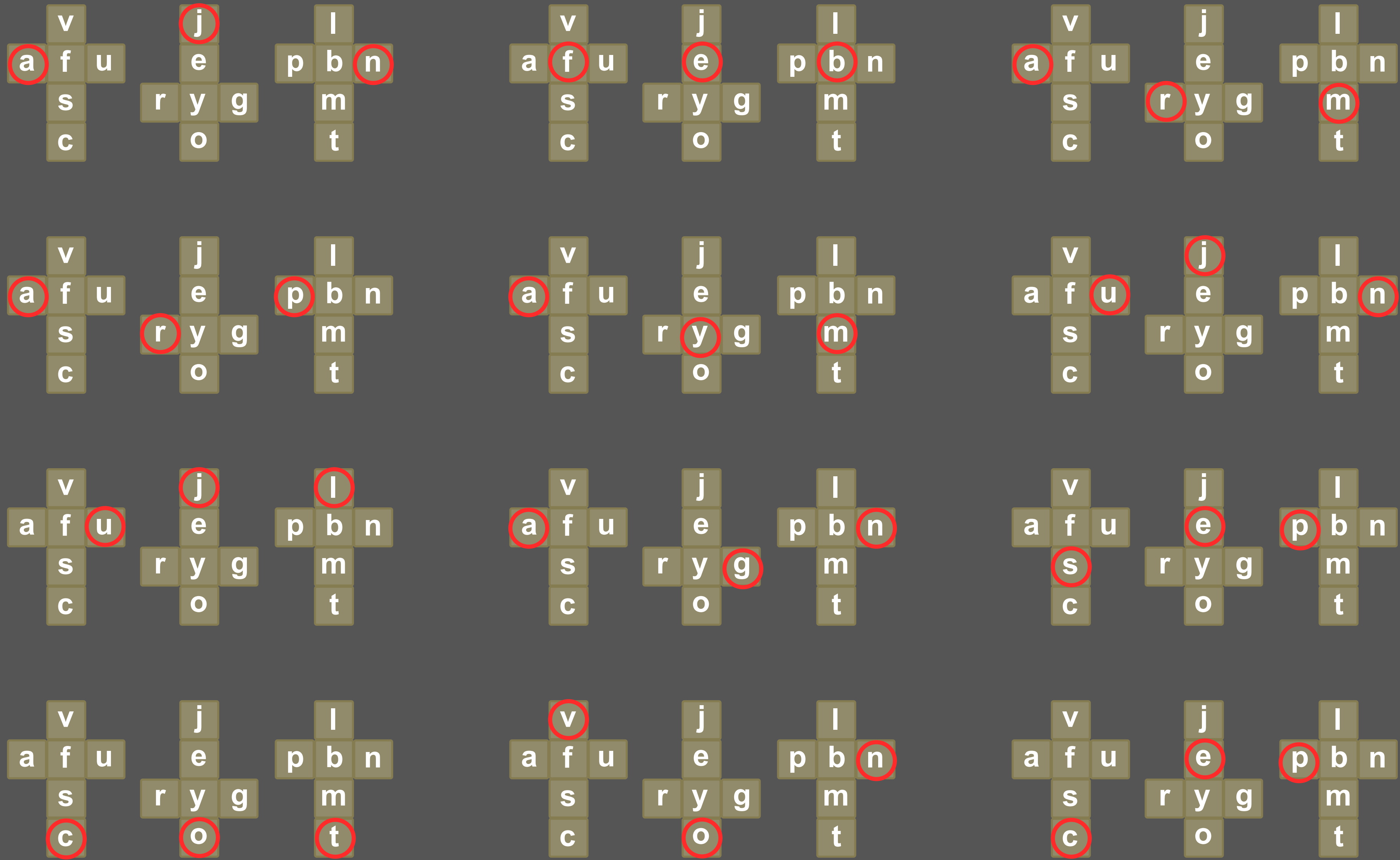
This is just one way of arranging the letters on the three cubes : perhaps you found a different arrangement? And perhaps you used a different approach? The following page shows how our arrangement can be used to give you all 12 months of the year.

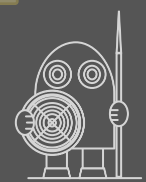
As mentioned in the question, you have to use a bit of cunning with some months : to get 'aug' you have to use the 'n' on one brick and turn it upside down – and to get 'dec' you have to use a 'p' turned upside-down.



PTO

ans 44 cube calendar - months



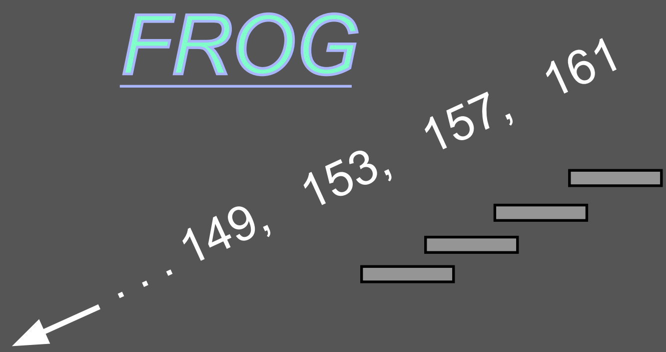


PTO

ans 44 cube calendar - months

. . . and finally, if you want to make a cube calendar of your own, you'll need five wooden cubes and some self-adhesive letters and numerals (Mr Pascal used Letraset Helvetica self-adhesive letters and numerals). Here's a layout which works :

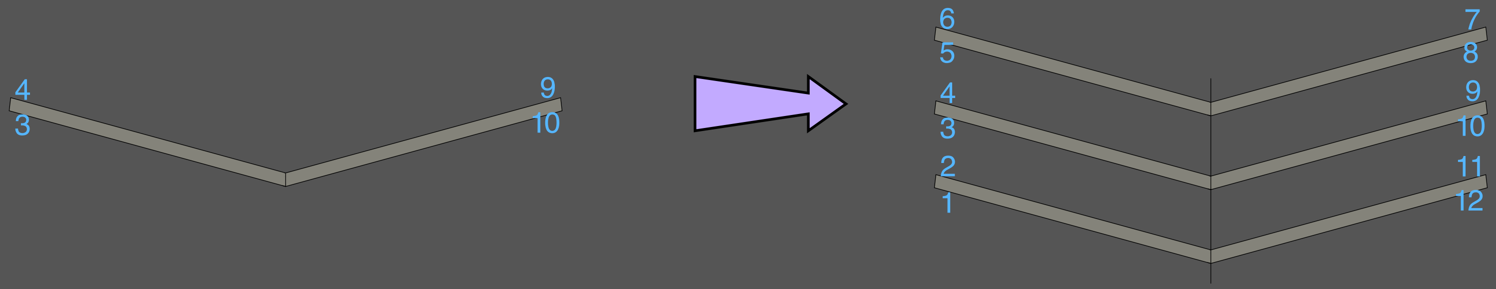
ans 45 stairway to heaven



The picture gives you an idea of how things are for the first few steps – the numbers show you clearly which step the ant (down at the bottom) and the frog (up at the top) are on at any point. Remember, the two of them start off at exactly the same time. Then when the clock chimes, they each jump. The ant jumps up by 3 steps each time; the frog jumps down by 4 steps. This means that each time they jump, the two of them get 7 steps nearer together. This tells you that that if the number of steps between them at any time is an exact multiple of 7, then the two creatures will surely meet. Luckily 161 – 1 = 160, which is not a multiple of 7. So . . . the ant seems likely to survive – at least for the time being . . .

ans 46 after the flood

One way of solving these two problems is just bit-by-bit to reconstruct the original booklets. It's easy to see with the 4 and 9 double-page, with 3 and 10 on the other side, that the sheet which used to lie underneath this one would have had 2 and 11 on the one side and 1 and 12 on the other side. And it's not hard to picture the remaining top sheet – with 6 and 7 on the one side, coupled with 5 and 8 on the other side :



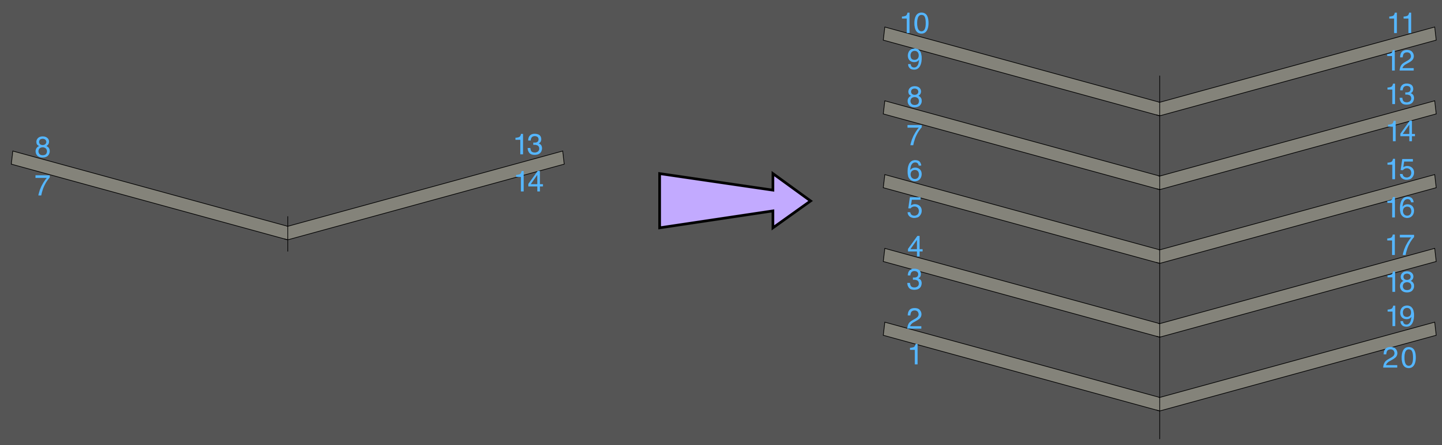
So, as you can see, there were 12 pages altogether in the original booklet – and the booklet was made from three single A4 sheets.

** Did you notice that the page-numbers facing you on any one sheet always add up to 13? Perhaps something to remember . . .*

PTO

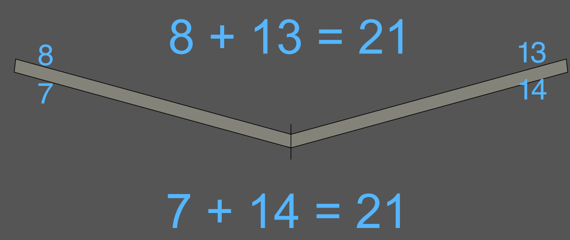
ans 46 after the flood

You can use the same approach with the other booklet. Start with the sheet you've got, with its four numbered pages, and work out what sheets (and what page-numbers) must have been above and below it :



As you can see here, there were 20 pages altogether in the original booklet – and the booklet was made from five single A4 sheets.

** Once again, the page-numbers on any sheet facing you always add up to the same thing – this time the total is 21 on every sheet. There's an obvious link between this total (21) and the number of pages in the booklet (20). Perhaps this would be a useful fact to know if you had to deal with a page from a much larger booklet . . .*



ans 47 action fractions

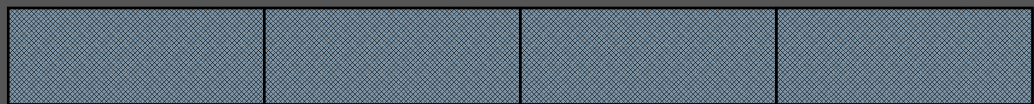
One way of getting to the answer is to use diagrams :

to begin with, let's use four boxes to stand for number 4, like this :

. . . and here's the action fraction a : 

next, this shows 4 (four boxes) minus the action fraction a :

but we're told that this is the same as 4 lots of a :



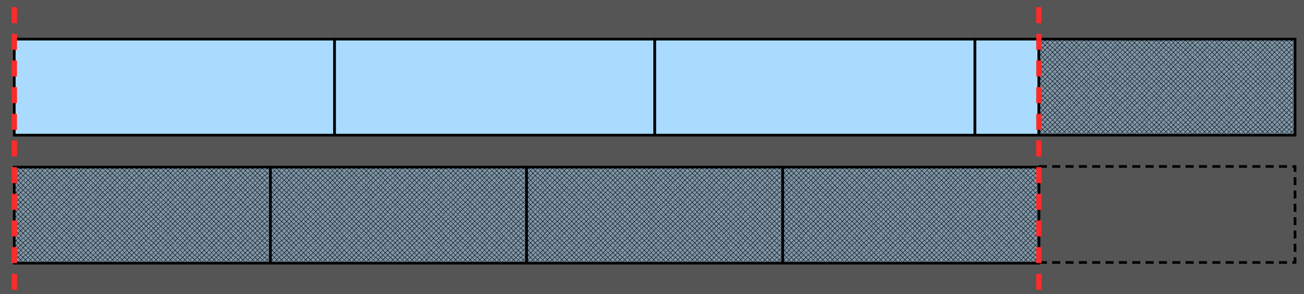
PTO

ans 47 action fractions

and we can show this fact by putting the two last diagrams together, as follows :

4 minus a :

4 lots of a :



* Now for the clever bit : The last diagram shows that 4 lots of a is exactly the same as 4 minus a.

We can also see that the diagram shows that 5 lots of a are exactly the same as 4,

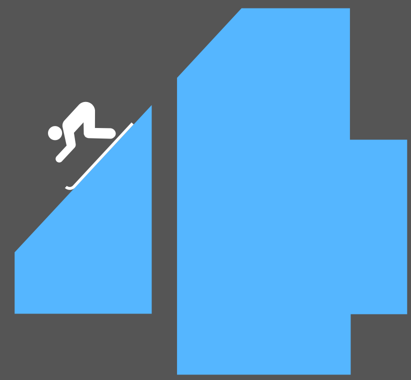
which of course means that a (the action fraction) must be 4 ÷ 5, that's to say 4/5.

So, the action fraction for 4 is exactly 4/5

CHECK : *4 – 4/5 = 20/5 – 4/5 = 16/5 . . . and 4 x 4/5 = 16/5*

PTO

action fractions



Another way of going about the problem is this :

to begin with, let's call our action fraction a

we're told that 4 minus a is exactly the same as 4 lots of a

which must mean that 4 equals 5 lots of a

and that's the same as saying that a = 4/5

so the action fraction we're after is 4/5

** if you know some algebra, you could show this way of getting the answer like this :*

$$4 - a = 4a$$

$$4 = a + 4a$$

$$4 = 5a$$

$$\underline{a = 4/5}$$

PTO ▶▶

ans 47 action fractions

If you'd like to do a little more work on this topic, here are one or two more ideas you could investigate :

4/5

4/5 is the action fraction for 4 (because 4 minus 4/5 is the same as 4 times 4/5 : they both come to $3\frac{1}{5}$) but

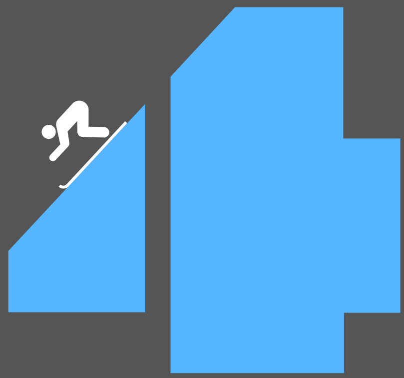
- What would be the action fraction for 2? or for 3? or for 5? or 6? There's a pattern here and it's not hard to find. How would you describe this pattern in words?

And if you're quite comfortable with fractions, you might like to try this (slightly harder) problem :

- Which fraction gives you the same result whether you subtract it from 1/4 or multiply it by 1/4 ? And what about other fractions like 1/2, 1/3 or 1/5? What's the action fraction for them? And what's the pattern?

PTO

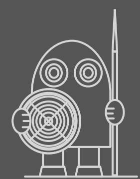
ans 47 action fractions



Well, here are our results – and as you can see, the patterns in each one are easy to describe. Of course, there are plenty of other interesting (and sometimes surprising) things to discover about fractions . . .

number	action fraction
2	2/3
3	3/4
4	4/5
5	5/6
. . .	. . .

fraction	action fraction
1/2	1/3
1/3	1/4
1/4	1/5
1/5	1/6
. . .	. . .



ans 48 trip saver

Perhaps one way to try to solve this is by trial and improvement . . .

First of all, something useful to note : Liss is saving in multiples of 2 and Khori is saving in multiples of 3. This means that whatever the amount is when they're at the same total, it will have to be a multiple of 2 and of 3 : in other words, a multiple of 6 !

So now let's start looking at some multiples of 6 and note how long each of them would take to reach this amount, remembering that Liss began saving in Week 1 and Khori began saving in Week 9 (that's 8 weeks later) :

£30

khori : 10 weeks at £3 per week

liss : 15 weeks at £2 per week

10 weeks and 15 weeks differ by only 5 weeks

£36

khori : 12 weeks at £3 per week

liss : 18 weeks at £2 per week

12 weeks and 18 weeks differ by only 6 weeks

PTO

ans 48 trip saver

£42

khori : 14 weeks at £3 per week

liss : 21 weeks at £2 per week

14 weeks and 21 weeks differ by only 7 weeks

£48

khori : 16 weeks at £3 per week

liss : 24 weeks at £2 per week

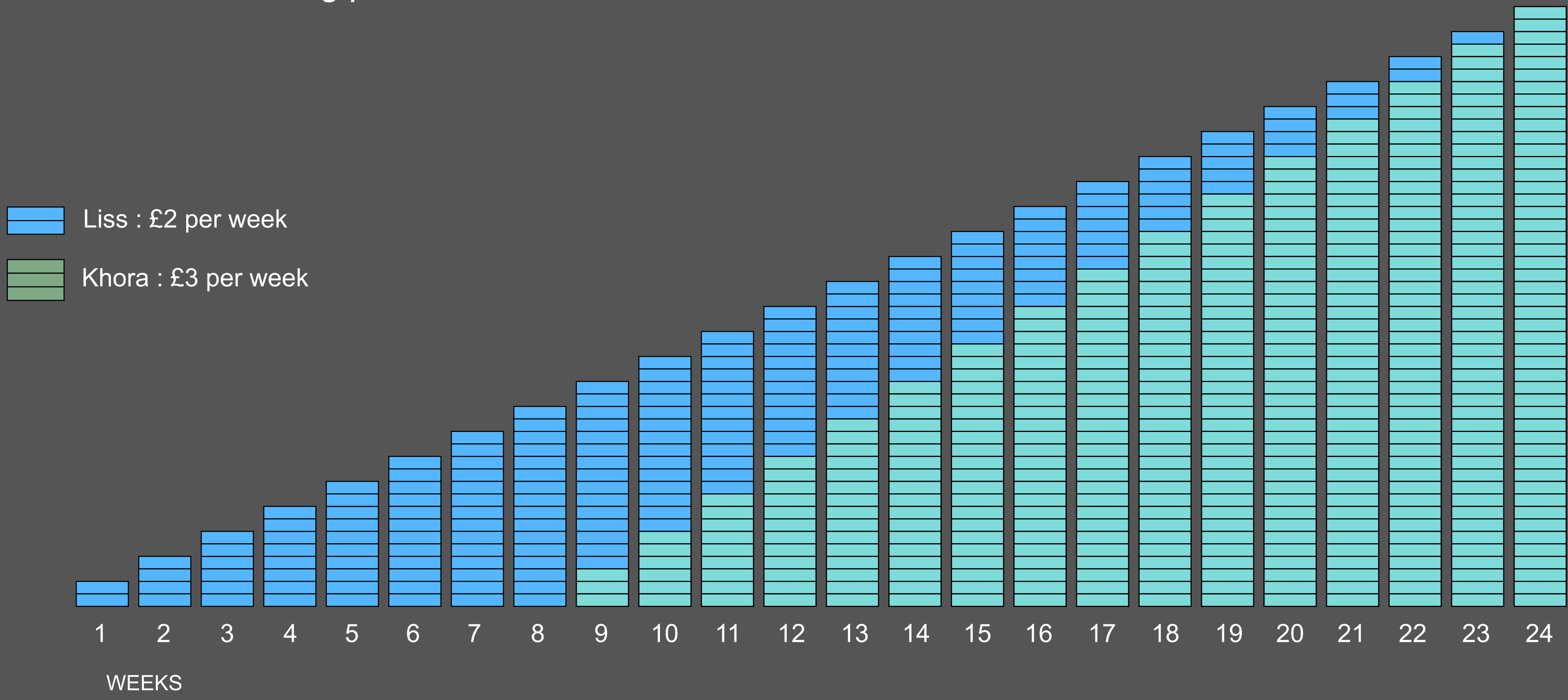
16 weeks and 24 weeks differ by 8 weeks — *and these are exactly the 8 weeks when khori hadn't started saving !*

So that's it! We've found that £48 is the amount at which the brother's and the sister's savings match – and we've found that this happens in week 24 of the year (that's Khori's 16th week of saving and Liss's 24th week of saving).

PTO ▶▶

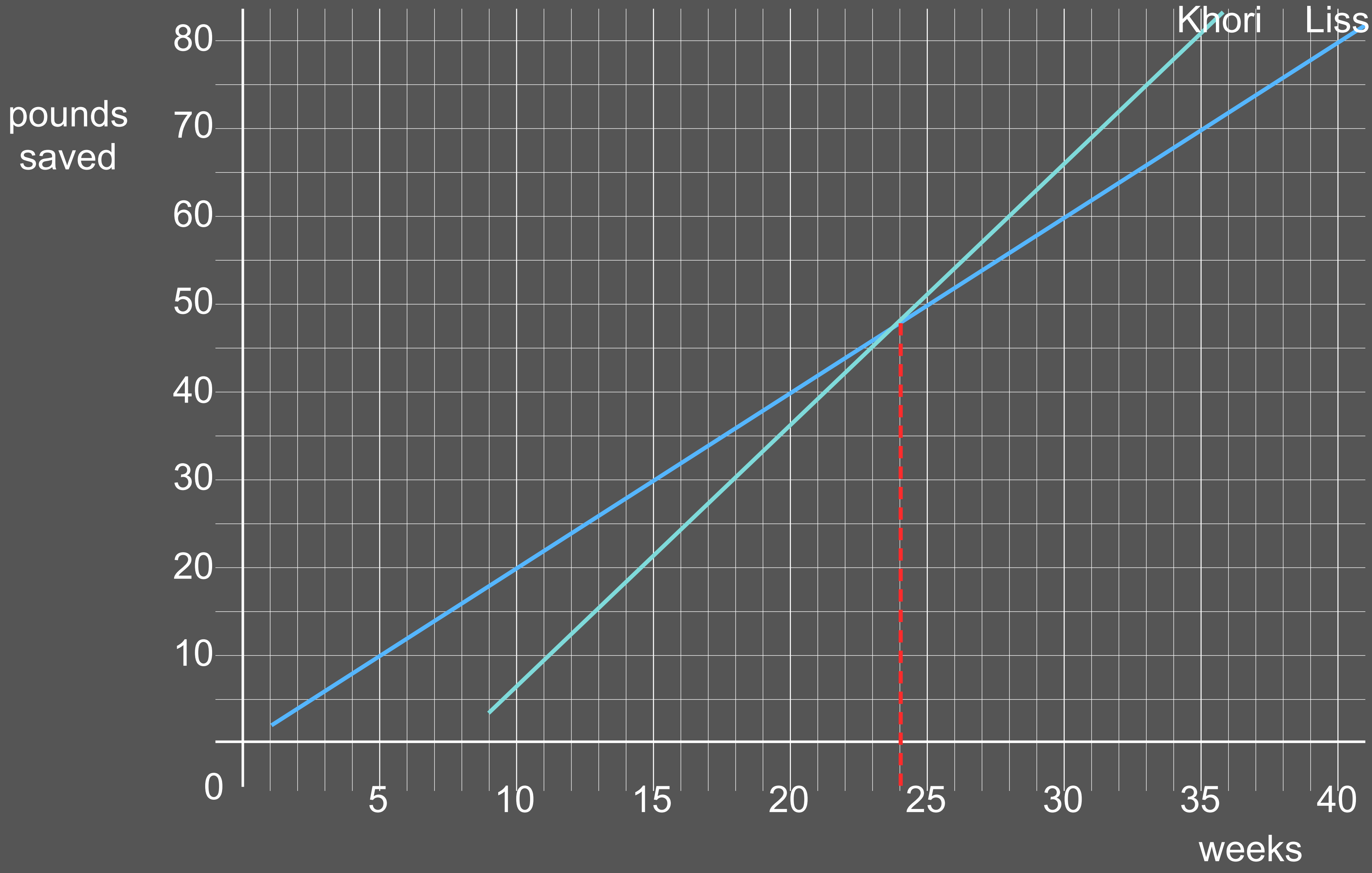
trip saver

Here and on the next page are two ways of showing the two saving patterns :



PTO

ans 48 trip saver

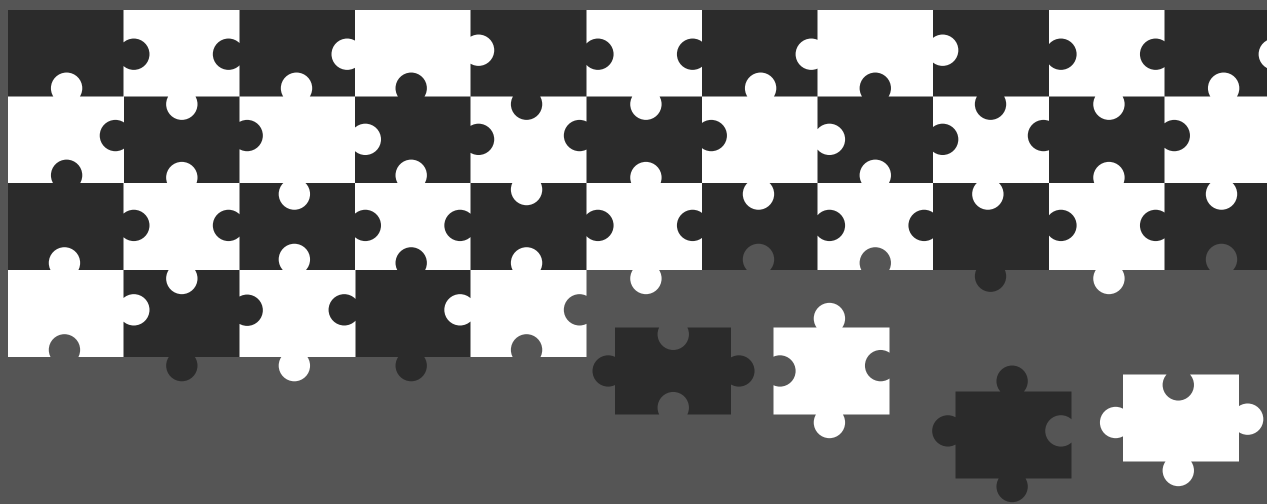


ANS 49 black & white jigsaw . . .

If you think about it, for a jigsaw to have one 'central piece', both its longer side and its shorter side must have an odd number of pieces. Syed's puzzle is 54 x 27, so as you can see, its longer side has an even number of pieces. This means that :

Syed's puzzle can't have one 'central piece'.

Now for the second question. Imagine putting this black-and-white jigsaw together. You would just lay down pieces along the top row, alternating black, white, black, white . . . and so on. Then you'd lay down pieces along the second row, alternating white, black, white, black . . . and so on. And you'd keep doing this all the way down to the bottom row, leaving the whole puzzle as a complete chequer-board of black and white :

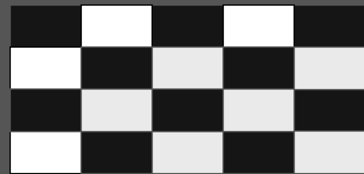


PTO

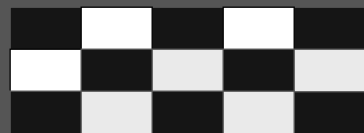
ans 49 black & white jigsaw . . .

And in fact, you'd find exactly the same number of black and white pieces in Syed's puzzle! But to see why this has to be true needs a bit of careful thinking. To begin with, we shall need to think about one or two different kinds of puzzle :

First of all, let's think of a puzzle with an even number of pieces in at least one direction. A simple 5 x 4 puzzle is one example. You can see straight away that the first **pair** of rows has 5 black and 5 white pieces – and so does the second **pair** of rows. Clearly, as long as we can split the puzzle into **pairs** of rows (or columns) like this, we'll always have the same total number of black and white pieces.



Next, we'll think of a puzzle with an odd number of pieces in each direction. A simple 5 x 3 puzzle is one example. As you can see, two of the rows have 3 black pieces and one row has 2 black pieces, making 8 black pieces in all. But only one row has 3 white pieces and two rows have just 2 white pieces, making 7 white pieces in all. It's clear that if we have an odd number of pieces in each direction, then there won't be the same number of black and white pieces.



The black-and-white version of Syed's puzzle does have an even number of pieces in one direction – so his puzzle would have exactly the same number of black and white pieces! And this time it's the columns you can group in pairs.

ans 50 match days count

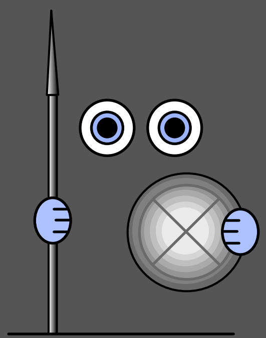
Look at the table below. Each column shows you one way of arranging matches for a league type of competition. As you can see, with 3 teams in a league you can organise things over 3 match-days (with 2 teams playing and 1 team resting on each day); and with 4 teams you can still fit the matches into 3 match-days (this time with all 4 teams in action on each match-day); with 5 teams you need 5 match-days.

3 teams		*4 teams*		*5 teams*	
	resting		*resting*		*resting*
A B	*C*	*A B / C D*	*–*	*B C / D E*	*A*
B C	*A*	*A C / B D*	*–*	*A D / C E*	*B*
C A	*B*	*A D / B C*	*–*	*A E / B D*	*C*
				A C / B E	*D*
				A B / C D	*E*
= 3 match-days		*= 3 match-days*		*= 5 match-days*	

There are different ways of arranging the matches, but for 5 teams you will definitely need only 5 match-days.

no
problem!
book?

answer book



four winds