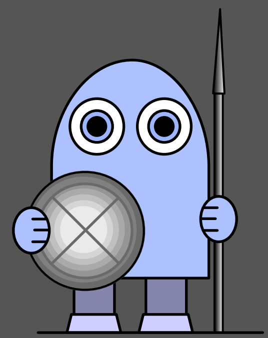
no problem ! book 2

50 maths problems with answers

four winds



four winds maths

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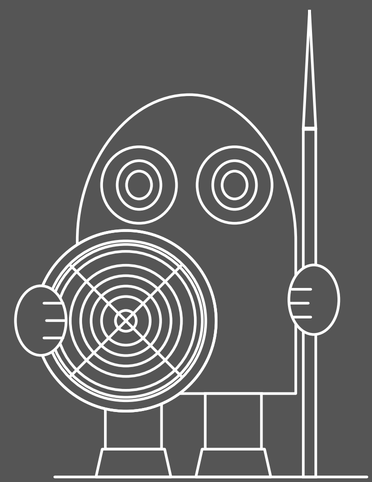


list of contents

1 your questions answered

Instead of a normal 'introduction' to the book, we've listed here our answers to some of the questions we have been asked.

2 list of problems

This is a listing of all the problems, colour-coded to show you which area of the subject each problem focuses on.

3 problems 1 - 50

This is the main part of the book : here are the problems themselves, starting with a few easier ones and then going on to the harder challenges, before finishing with a small number of really quite difficult ones.

4 answers 1 - 50

Here are full answers and explanations for all the problems. There's no right way of solving any of them and so for many we show you different ways of tackling them. This is purely for your interest of course ~ we know that for any of these problems, you might well have a better way (one which works well for you).

your questions answered . . .

We hear about 'problem-solving' all the time but what exactly do you mean by it?

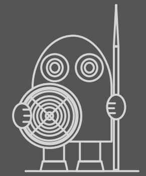
Imagine you're in the maths class. The teacher says,'I want you to do these problems' and then she writes on the board a dozen long multiplications : 23 x 17, 46 x 54, 38 x 76 . . . and so on. Well, we wouldn't really call these 'problems'. As long as the class has already learned how to do long multiplication, what's written on the board is really just a set of questions, or an exercise if you like. You've probably done quite a few things like this in your time.

What makes something a problem is this : when you read the thing through, you don't immediately know how you're going to get to the answer. You understand the question, perhaps you know what sort of answer you're going to get – but because what you're being asked to work out is new to you, you have to use your imagination, you have to be creative, if you're going to find a way of getting to the answer.

The other thing about problem-solving which you'll soon find is that the problems are often written out in words; in many problems there's a story about something or other and at the end of it there's something you're asked to work out. You have to read the question very carefully to be sure of the maths facts in it. Of course not all problems are like this; for example, many shape/space problems start with a diagram which you have to look at carefully.

We've given an example of questions which wouldn't count as problems. Here's an example of something which would count as a problem :

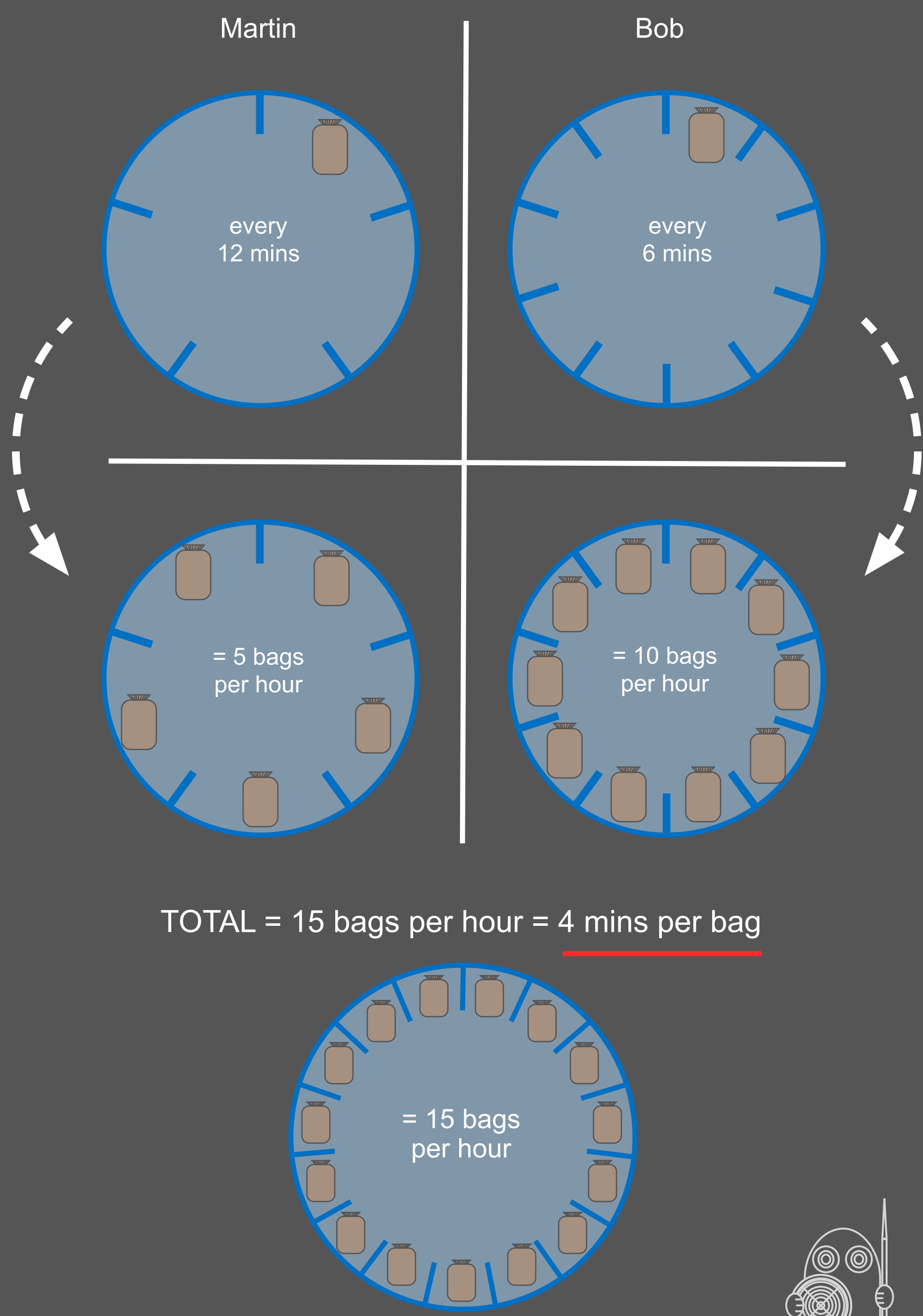
Every Saturday, Martin and Bob work in a chip shop. As you can imagine, there are lots of potatoes to be peeled and neither of the boys enjoy doing this. Martin can peel a bag of potatoes in 12 minutes but Bob can peel a bag of potatoes in 6 minutes. How long would it take the boys to peel a bag of potatoes with both of them working on the same bag?



Imagine we're faced with this problem and we've already tried various things without any success. What shall we do next? Well, one thing we can do is to see if there's a way of maybe looking at what we've been told in a different way. Suppose we look at our information the other way round, that's to say don't look at how long it takes each boy to peel a bag of potatoes – instead, look at how many bags per hour each boy can peel . . . So, now our information reads like this : Martin can peel 5 bags of potatoes in an hour and Bob can peel 10 bags of potatoes in an hour. With things written this way round it's easy to see how we can combine what the two boys do : together they can peel 15 bags of potatoes an hour (5 bags plus 10 bags). And 15 bags per hour is the same as 4 minutes per bag. Now we have our answer : together the two boys will take just 4 minutes to peel a single bag of potatoes.

answer : 4 minutes

** Notice that, as with all maths problems, we had some starting info and we knew what sort of answer we were after (in this case, a number of minutes – and definitely less than 6 minutes).*



How are maths investigations different from maths problems? With both of them you seem to start with something that's unknown and from there you have to find your way to an answer.

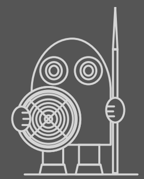
These two things are similar but *. . . with a maths problem you start with some facts about a situation and then you have somehow to find out a different fact. You probably know what* kind *of answer you're looking for : how old someone is or how long a journey takes or what the area of some shape must be – or whatever. But with a real problem there will be something about it which is unfamiliar to you – and that's why at first you won't know how you're going to go about things. You'll have to think of a way of using the facts you're given to get to the answer you want.This is what can make problem-solving hard . . . tbut it's also what makes problem-solving interesting.*

With a maths investigation, on the other hand, you begin with a situation or set-up (maybe a particular shape or shapes or a set of numbers or whatever) and a rule, or a set of rules, for doing somehing to this original set-up. You apply the rule / rules and see what changes you get. Perhaps as a final step you try to find a pattern in your results or some other feature which you can describe neatly – and that's the outcome of your investigation.

With a maths problem you don't know at first how to get to the answer – but you do know what kind *of answer you'll have in the end. With a maths investigaton you don't even know what kind of results you'll end up with; you just know where you're starting and what rules to apply.*

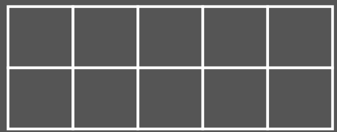
There are all sorts of things in maths just waiting to be investigated – and there are many kinds of investigation. But on the following page we've given you just one example. It's a fairly straightforward maths investigation called 'Rectangle Crossings' and here we've described it only briefly. If you go through this carefully, you'll see pretty clearly the features of a 'maths investigation' which we've described above . . .

ps If you want to read more about this and other maths investigations, – and most of all, if you want to try some maths investigations yourself – you'll find plenty to interest you in 'Maths Investigations', which you can download free of charge from our website, www.fourwindsmaths.com

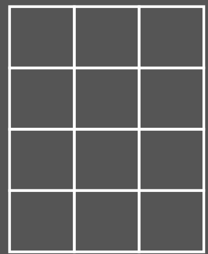


Rectangle Crossings

starting point : *We begin with a number of rectangles made up of unit squares, like these two :*



5 x 2

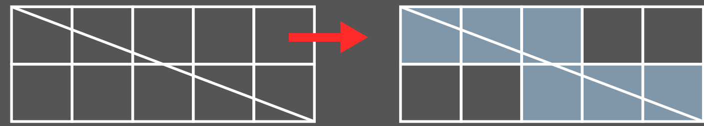


3 x 4

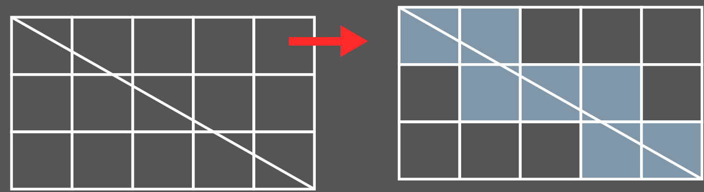
For the moment, we'll stick to rectangles where the two sides don't have any factors in common. So the 3 x 5 rectangle, the 7 x 2 rectangle, the 6 x 2 rectangle, and the 8 x 3 rectangle are other examples. You get the idea . . . Because we've used unit squares in the making, the sides of these rectangles are all whole numbers. Now we know rectangles like these are our starting-point, let's move on to the rules for carrying out the investigation :

instructions : *For any rectangle you're investigating, draw a single line (or diagonal) to join two opposite corners of the rectangle; next, count the number of squares which the diagonal crosses ('crossing a square' means 'cutitng across the square', so just touching the corner of a square definitely doesn't count). Make a table to show your results and finally – look for a pattern in these results.*

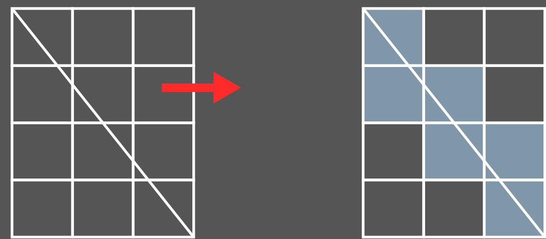
results :



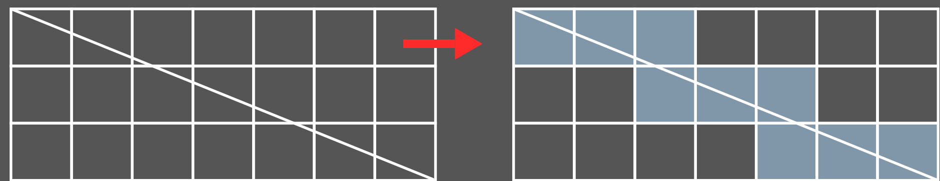
5 x 2 *crosses 6 squares*



5 x 3 *crosses 7 squares*



3 x 4 *crosses 6 squares*



7 x 3 *crosses 9 squares*

conclusion : *The pattern hidden in our results is not hard to find. Look at each of the rectangles and try adding up the two sides. What do you get? You always get a number which is 1 more than the number of sides crossed. So now we can confidently say that the pattern our investigation has produced is this :*

To get the number of squares crossed by a diagonal, just add the two sides and subtract 1.

extension : If you're feeling brave, you might like to investigate what happens when we have rectangles like 4 x 4 or 6 x 2 or 3 x 9 or 5 x 5 , where the two sides do have one or more factors in common (other than 1). Mathematicians call such pairs of numbers 'relatively prime'. You'll soon see that the simple pattern written out above doesn't apply to rectangles where the sides are relatively prime. In case you do want to investigate these rectangles, we won't show the pattern here – but, for your interest, it's noted at the end of this section.

What's the difference between a problem and a puzzle? And what's the difference between a maths problem and a maths game?

When you've solved a maths problem, you've probably increased your understanding of the subject in a wider way – which means you'll be able to apply something of what you've learned to other maths situations. Whereas a puzzle usually has a specific solution – or in other words, there's just one way of solving the puzzle and often the solution amounts to a particular 'trick', that's to say not something which you can apply more generally . . .

When it comes to maths games, you probably know that there are different sorts. There are 'games' for just one player (such as Square Shuffle or Last Kangaroo) : often these are more like puzzles. But there are also some really good games for two players (such as Alquerque or Circle of 8). With these your'e aiming to beat the other player and to do this you have to understand how the game works, so you can develop a winning strategy. This is very much like the problem-solving we've been talking about.

I want to become good at problem-solving. What do I need to do?

The best way to become good at problem-solving is to do lots of problems. You might buy a book of problems (like this one) and start working through it. At first each question really is a problem to you and you have to think hard and probably try lots of things before you hit on the best way to solve it. But gradually you'll begin to recognise problems that are like ones you've seen before and managed to solve. Now life begins to get easier and you could say that many of the questions are not problems any more (at least, not to you). So in a way what counts as a problem and what counts as a more or less routine question depends on whether you've seen something like it before.

ps Thinking just about number-based problems, you'll find that certain sets of numbers come up quite often. So if you're in this thing seriously, it's a good idea to at least make sure you're familiar with these sets :

the prime numbers (2, 3, 5, 7, 11 . . .)

the odd numbers (1, 3, 5, 7, 9 . . .)

the even numbers (2, 4, 6, 8, 10 . . .)

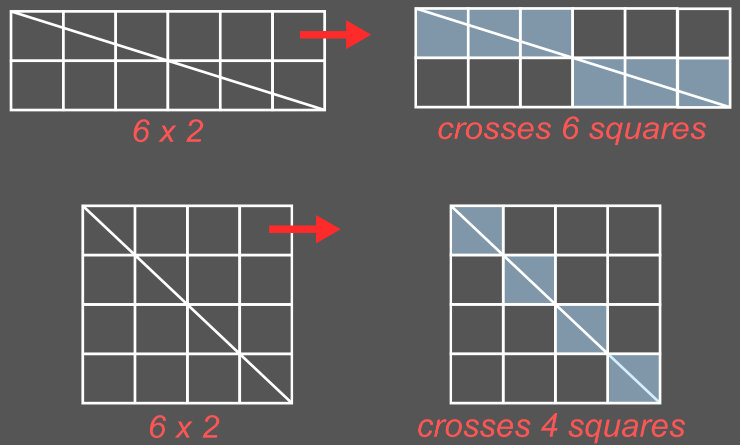
the square numbers (1, 4, 9, 16, 25 . . .)

the triangle numbers (1, 3, 6, 10, 15 . . .)

If you want to know more about these sets of numbers, there's plenty of information on the internet – or you can read our book on sequences (called 'one thing after another', due out in 2022/3 and full of interesting information and helpful explanations. Many young mathematicians have found it useful to learn off by heart eg the first ten prime numbers, the first twenty square numbers etc. And don't be afraid! If you take these sequences one at a time, it's much easier than you think to learn the first few numbers in each, especially if you've got someone who can say them with you and then keep testing you until you've got them under your belt . . . after all, compared with other subjects, maths really doesn't have many things you need to learn off by heart.

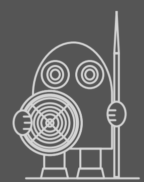
Rectangle crossings final note
– rectangles with relatively prime sides

examples :



the new rule : *Our first version of the pattern applied only to rectangles with two sides relatively prime. If we want a pattern which applies to all rectangles, even those where the two sides have at least one factor in common (other than 1), we need a more sophisticated version :*

To get the number of squares crossed by a diagonal, just add the two sides and subtract the largest common factor.



list of problems

1 mark time
2 shopping day
3 mean cyclists
4 bottle-tops are go
5 japanese magic circles
6 the job's yours
7 spaced-out kids
8 jane's people
9 flag day
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18 completely nuts
19 annabelle sails
20 dockyards & warships
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26 sum of three primes
27 transport for london
28 marking time
29 fault lines
30 an open and shut case

PTO

KEY

problems on logic, sets, combinations, permutations, probability, statistics

miscellaneous number problems (based on pre-algebra skills)

problems involving various aspects of shape (2-D and 3-D)

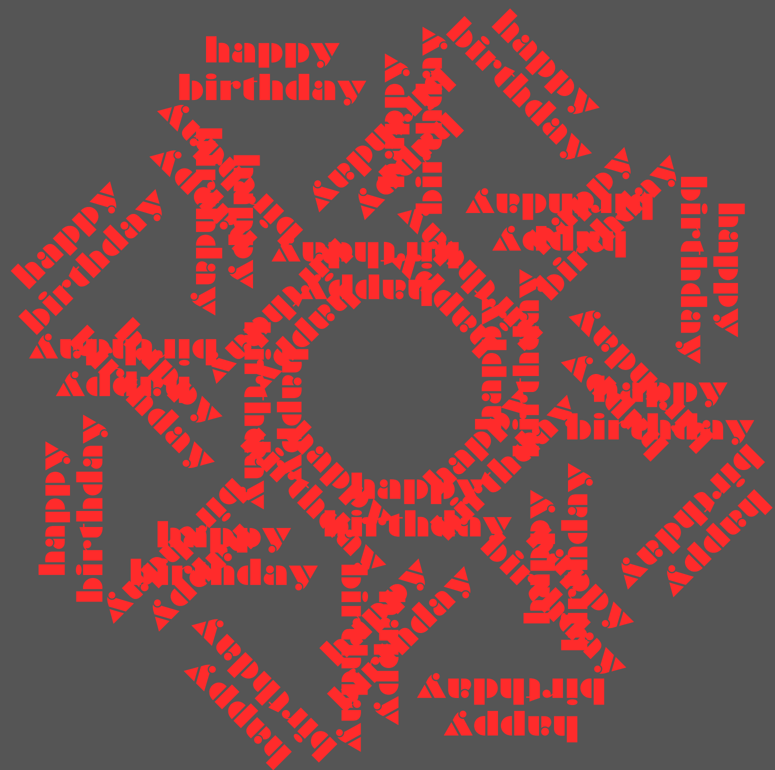
mark time . . .

Susan and her younger brother Mark both have their birthdays today. Here are two facts about their ages :

- Today Susan is three times as old as Mark.

- In two years' time, Susan's age will be exactly double Mark's age.

From these two facts you can work out everything you need to know about the ages of the brother and sister. Your problem is this : how old will Susan and Mark be in two years' time?



2 shopping day

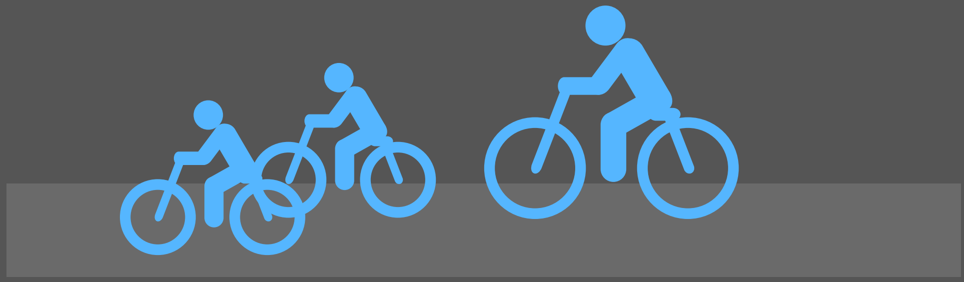
Last week it rained on Monday, Tuesday and Wednesday; it was sunny on Thursday and Friday – and then on Saturday and Sunday it was cloudy. Lucy went shopping on just one of the days last week and Sam went shopping on another day.

Here are a few facts :

- it was raining when Sam went shopping
- Lucy went shopping the day after Sam
- it wasn't raining when Lucy went shopping
- neither Lucy nor Sam ever go shopping at weekends

QUESTIONS : On which day did Lucy go shopping? And what about Sam ?

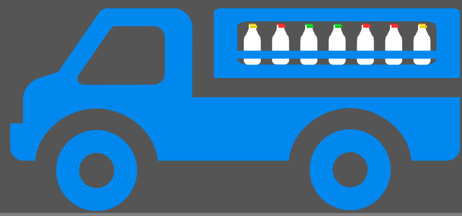
3 mean cyclists



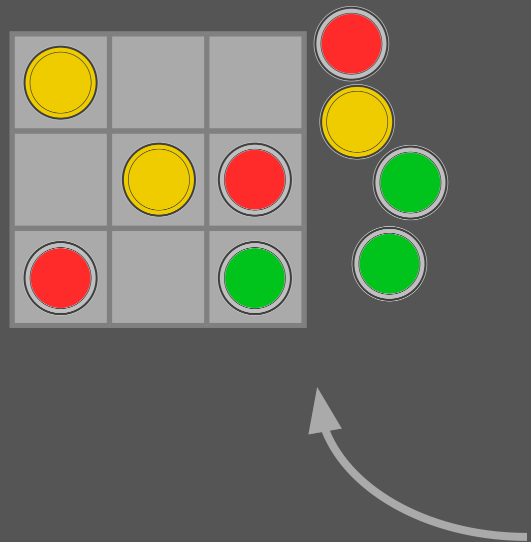
There are three children in the Evans family : older brother Roger and sisters Emily and Sarah. Here are some facts about their ages :

- Sisters Emily and Sarah are actually twins.
- The mean of all three children's ages is 7.
- The twins are 6 yrs old.

How old is Roger?



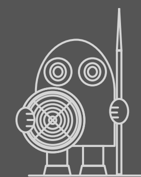
4 bottle-tops are go!



In some parts of Wales people still have their milk delivered each morning by the local milkman.The Morgan family (2 adults and 5 children) are just like this. Each day the milkman leaves them 9 bottles of milk : three are red-top bottles (ordinary milk), three are gold-top (extra-creamy milk) and three are green-top (low-fat milk).

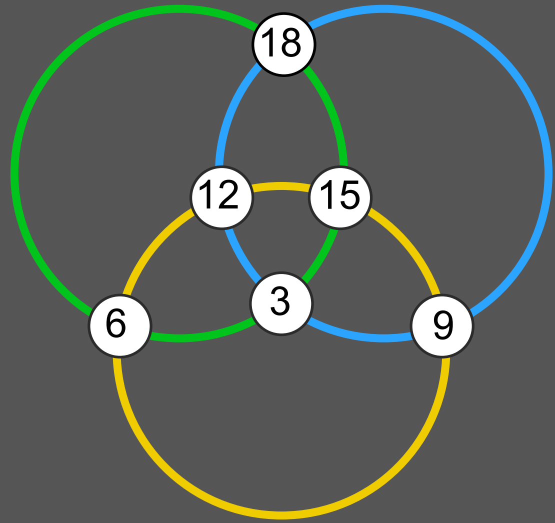
Here's a diagram showing you what the Morgans get each day. Can you arrange the bottles in the crate so that the coloured tops form a Latin Square? And, with these same colours, how many Latin Squares can you make?

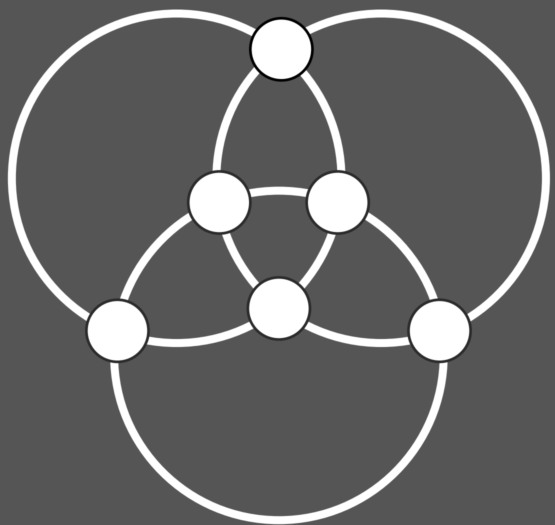
You might not have come across Latin Squares before but the idea is a fairly easy one. A Latin Square is a square arrangement of things (numbers, colours, letters or whatever you like) with a simple rule : you can't have the same thing twice in any row or column. Latin Squares can be 3 x 3 or 4 x 4 or 5 x 5 – or any other size.



5 Japanese Magic Circles

Look at the diagram on the right. To start with, just look at the green circle. If you go round the green circle adding up the numbers which lie on it, you get 18 + 15 + 3 + 6, which equals 42. Now suppose you go round the blue circle adding up its numbers; you get the same total, 42. And with the yellow circle you get exactly the same total. Arrangements like this are called 'Japanese Magic Circles'.



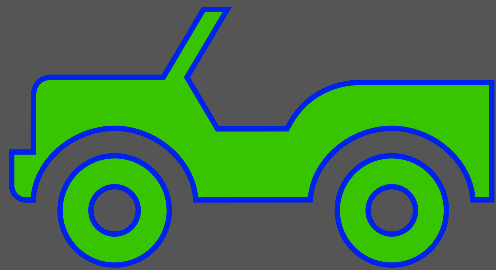


your problem : First of all, make a quick sketch of the diagram on the left : to do this, just draw a set of three overlapping circles (no need for colour) and draw little circles at all the places where the big circles cross. Next, find a way of arranging the numbers 13, 7, 4, 11, 8, 2 in the small circles so that you get the same total whichever big circle you travel round.

6 the job's yours !

‘The Major’ (a well-known criminal mastermind) is planning his next robbery. He’s got a team together but he’s still one man short. He needs someone who’s a good driver, can handle guns and is good at disguise. He has four men to choose from : Jake, Sam, Pete and Frankie. This is what we know about them :

- Jake and Sam are known to the police
- Jake, Sam and Frankie can drive
- Jake and Frankie are good safe-breakers
- all except Frankie can handle guns
- all except Jake are good at disguise
- both Jake and Pete know the area well



Which man will The Major choose to join his team?

7 spaced-out kids

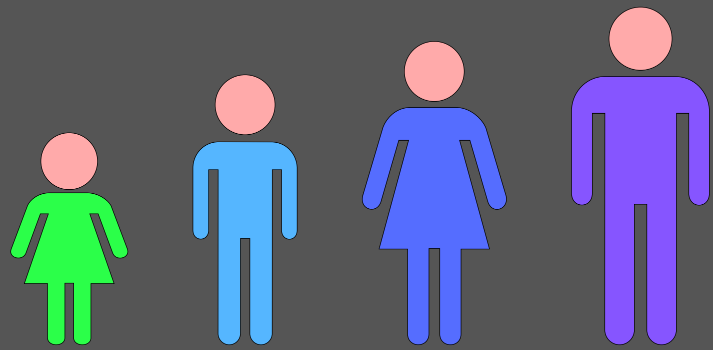
There are four children in the Pascal family; they are Anna, Bertrand, Christine and Daniel. Here are some facts abut their ages :

Anna is the youngest

Bertrand is 2 years older than Anna

Christine is 2 years older than Bertrand

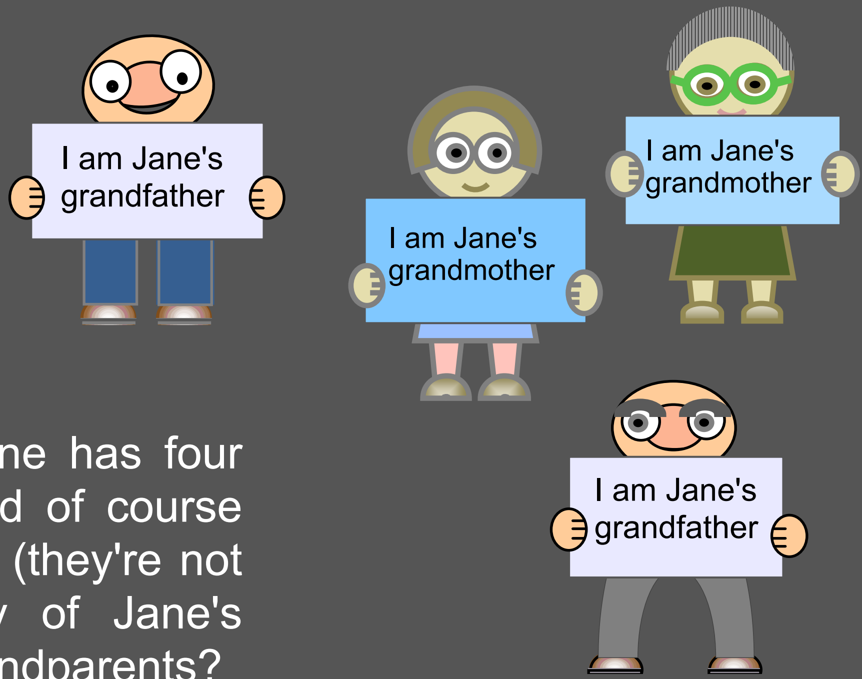
Daniel is 2 years older than Christine



If you add up the ages of all four children, you get a total of 28.

Your problem : how old is Bertrand?

8 Jane's people

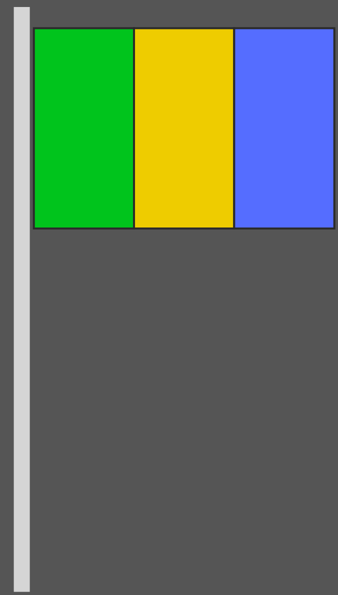


Jane and Mary are cousins. Jane has four grandparents (pictured here) and of course Mary also has two grandparents (they're not pictured here). But how many of Jane's grandparents are also Mary's grandparents?

9 flag day

One day, Mrs Robinson got her twenty-four Reception Class children to design flags. Her instructions were :

1. *Each flag must have three stripes.*
2. *You must use three different colours for your three stripes.*
3. *You must use blue, green and yellow as your three colours but you may arrange them in any order you like.*



On the right is a picture of one of the flags the children made. As you can see this one has green, yellow and then blue for its three stripes.

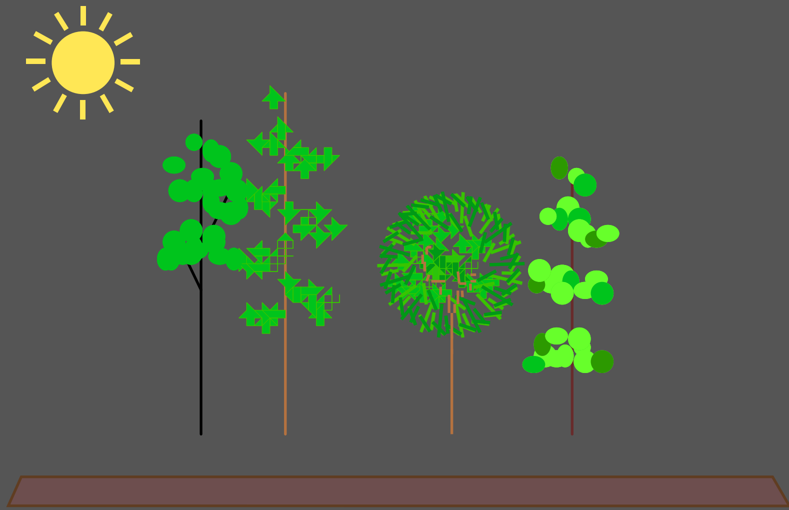
When the children had finished, Mrs Robinson saw that they had indeed made flags using different arrangements of the three colours.

- *How many different arrangements are possible, using each of the three colours once and once only ?*
- *If there were equal numbers of all possible arrangements, what's the probability that a flag picked up at random has a blue stripe between two other colours ?*

10 it's a clean sweep!

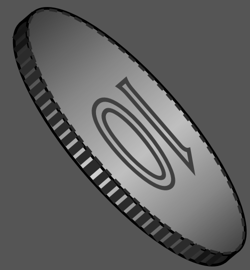
Billy and Joseph (his older brother) spend most of Saturday afternoon sweeping up leaves and generally tidying the garden for their neighbour, Dr Oliver. At the end of their labours, Dr Oliver pays them £15 for all they've done. Joseph says that as Billy is quite a lot younger than he is and also because Billy doesn't sweep up as many leaves, he won't share the £15 equally. Instead, says Joseph, he will be getting £3 more than Billy.

With this way of doing things, work out how much will be given to each of the boys.

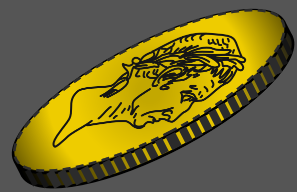


heads and tails

Every coin has two sides : a head and a tail. Jake has an ordinary 10p coin in his pocket. He takes it out, flips it into the air and catches it on the palm of his hand. What's the probability that the coin has landed head-side up? As you probably know, the answer's 1/2. (That's because if you flip the coin often enough, it will land heads-up about half the time.)



Jake's sister Annabel has two older coins, a 10-cent coin (silver) and a 20-cent coin (bronze), both exactly the same shape, size and weight. She flips them both into the air together, then skilfully catches them in the palm of her hand.



- What's the probability that when Annabel looks at the coins, she finds they're showing 2 heads?
- What's the probability that when Annabel looks at the coins, she finds they're showing a head and a tail?

NOTE : In case it isn't clear, the 'head' sides of Annabel's coins are the sides with the image of a head on them – and the 'tail' side of her coins are the sides with either a '10' or a '20' showing.

12 will there be time ?

You probably know what a palindrome is – it's something which reads the same from left to right as it does from right to left. In other words, you get the same result whether you go through the thing backwards or forwards.

PALINDROMES CAN BE

single words eg names like anna or otto

phrases eg never odd or even

whole sentences eg was it a car or a cat I saw?

numbers eg 2734372

Your problem : As you know, a digital clock always has four figures in the display. Now, if you look at the clock on the right, you'll see that the time it's showing make a palindrome. Try to find all the palindromes this clock will show in each 24-hour period.

13 mean Mr Francois

Mr Francois is a very angry man. In fact, last week the average (mean) for Mr Francois getting angry was 7 times per day. How do we know this figure? Well, last week some of his pupils noted each time he got angry and then at the end of the week they calculated the mean in the usual way. The figures we have for each day are as follows :

Monday: 4 times, Tuesday: 8 times, Wednesday: 7 times, Thursday: 6 times, Friday: ?

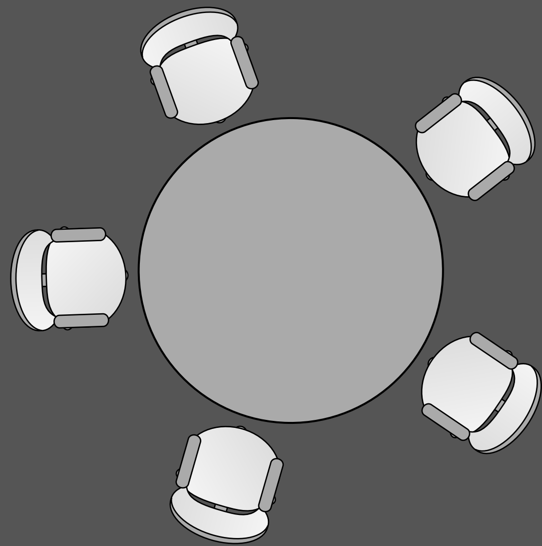
When Mr Francois gets angry his eyes bulge and his face turns a sort of dark red; he also shouts and waves his arms about rather a lot. It's a frightening thing to see. But the figure for Friday is missing from the table above. What should it be? How many times did Mr Francois get angry on Friday? That's your problem . . . anger is his problem.

ps He pronounces his name 'Frarn-swa'

14 come round for a meal !

Five friends – Alfred, Beatrice, Charles, Diana and Ellie – are having a meal together. Here's some information about exactly where they are sitting (it's a round table, by the way) :

- Beatrice is sitting between Charles and Diana
- Alfred is sitting on Ellie's right
- Diana is sitting on Alfred's right



Try to work out who is sitting where and then answer these two questions :

1 Who is sitting on Ellie's left?

2 Who is sitting on Diana's right?

15 digit-sums

What exactly are 'digit-sums'? Have a look at the mappings below and you'll soon get the idea :

421 -> 7

435 -> 12

455 -> 14

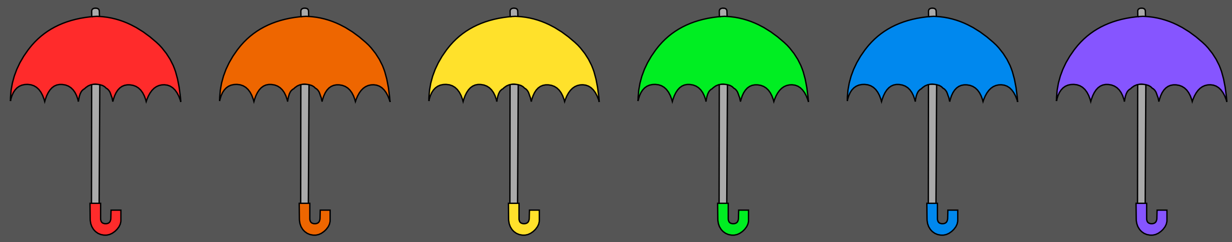
470 -> 11

Can you find a simple connection between the three-digit numbers shown on the left and the numbers shown on the right ?

You've probably seen that if you add up the digits of any number on the left, this total (called the 'digit-sum') is what's shown on the right . . . Now answer these :

- what's the smallest digit-sum from numbers over 400 but under 500 ?
- what's the largest digit-sum from numbers over 400 but under 500 ?
- how many numbers over 400 but under 500 have a digit-sum of 12 ?

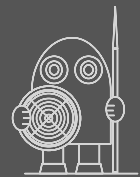
16 singin' in the rain



At the end of a very wet year, the Manhattan Umbrella Shop looked back at which umbrellas had sold well and which had sold poorly. They counted the sales of different colours and then they listed all the colours in order. Here's what they found :

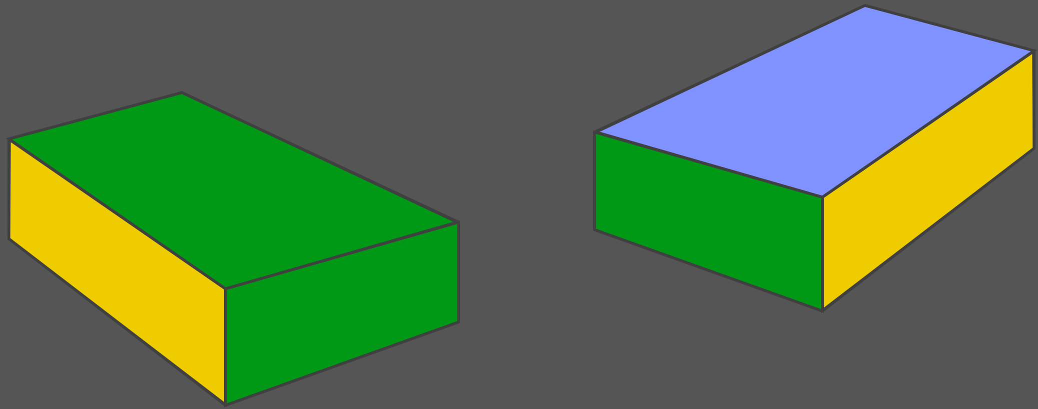
- yellow and purple sales were equal
- green came above orange
- there were three colours between orange and blue
- there were 50 more yellow umbrellas sold than orange ones
- red topped the poll – more red umbrellas were sold than any other colour
- there were two colours between blue and green
- blue was a close runner-up; it almost came first

Using the information above, work out the positions of all six umbrellas in the final list.



17 no red faces here !

Here are two different views of the same box :

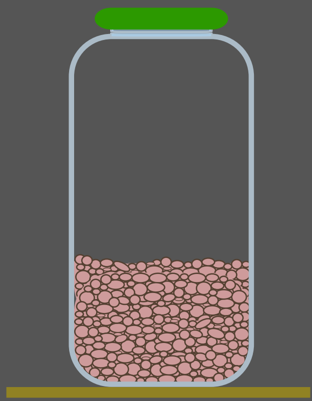


- there are two green faces
- only one of the two smallest faces is green
- the blue face you see doesn't have an edge in common with any other blue face

your question : considering the facts above and looking at the two views of the box, can you work out how many yellow faces there must be ?

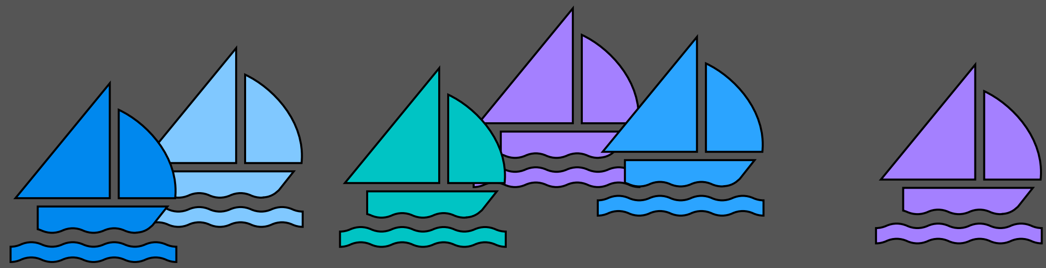
18 completely nuts !

On Boxing Day, there's a jar full of nuts next to the Christmas tree. The jar and the nuts together weigh 1.225 kilograms. A few days later, exactly half of the nuts have been eaten; now the jar and nuts together weigh just 784 grams. By New Year's Day the jar is empty.



What does the empty jar weigh ?

19 Annabelle sails



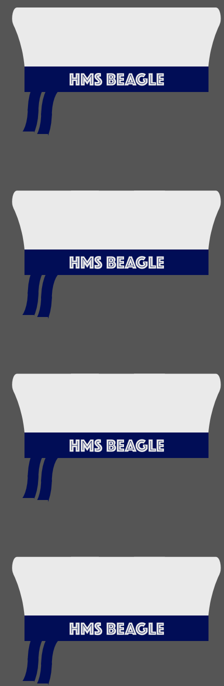
Annabelle loves sailing. Last weekend she took part in the annual Roxburgh Regatta in Nevada, USA, where the main event was a two-part race across Lake Hubron. On the Saturday, competitors had to sail straight across the lake from Roxburgh to Gadlas Creek – and on the Sunday, competitors had to sail the very same course in the opposite direction. Each day's race began at 11:00 exactly. The distance from Roxburgh to Gadlas Creek is exactly 18 kilometres, by the way.

On the outward journey, Annabelle sailed past a small island. Next day, on the race back to Roxburgh, Annabelle sailed past this island once again. The strange thing was that she passed the island at exactly the same time each day. The other thing to report is that on the outward race, Annabelle sailed twice as fast as she did on the return race. (Probably something to do with the prevailing wind.)

How far is the small island from Roxburgh?

20 dockyards & warships

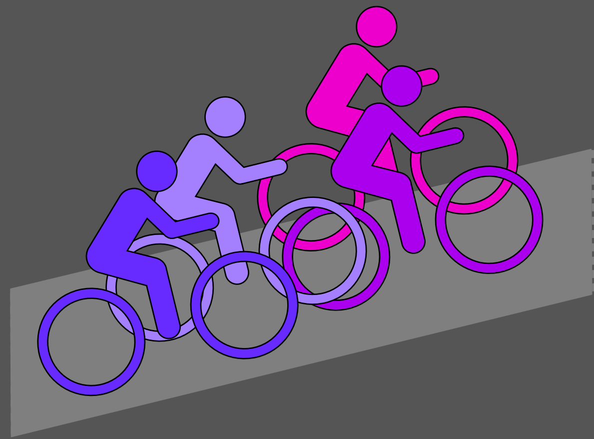
One day in Spring, a small naval patrol vessel, the HMS Beagle, sets off from Plymouth. It's bound for Rosyth in Scotland and the four sailors on board are looking forward to the trip. The names of the sailors are Anson, Byng, Cavendish and Douglas. There are various things on the vessel which must be attended to at all times, such as steering, keeping a lookout, maintaining signals and so on. To do these things properly, there must be three sailors on deck at any one time.



Your problem is this : How many different groups of three can you make up from these four sailors?

21 uphill & downhill

The Four Peaks Cycle Race takes place on the first Sunday in May each year. This is one of the most famous outdoor cycle races in the country and hundreds of brave cyclists, both men and women, take part. It's a difficult race, featuring some really tough uphill stretches and some quite giddy downhill ones. (There are no level stretches.) Here's the official description of the course :



"THE COURSE : The race begins at Belvoir Point (743 metres above sea-level) and from here climbs steeply up to Carr's Peak (height 982 m), before a shorter section takes the riders down to Denton Dene (height 823 m). Next follows the longest climb of the whole race, which takes the riders up to Eagles' Nest (height 1253 m). After this come short downhill and uphill sections, taking the riders first to Fallon Peak (height 1107 m) and then to Glyn Mount (height 1261 m). Next comes the longest downhill section of the race, which leads from Glyn Mount to Home Hollow (height 475 m). The race ends with a slightly easier climb to Ingle Peak (height 687 m) and a short downhill descent to the finish at Juniper Road (height 521 m)."

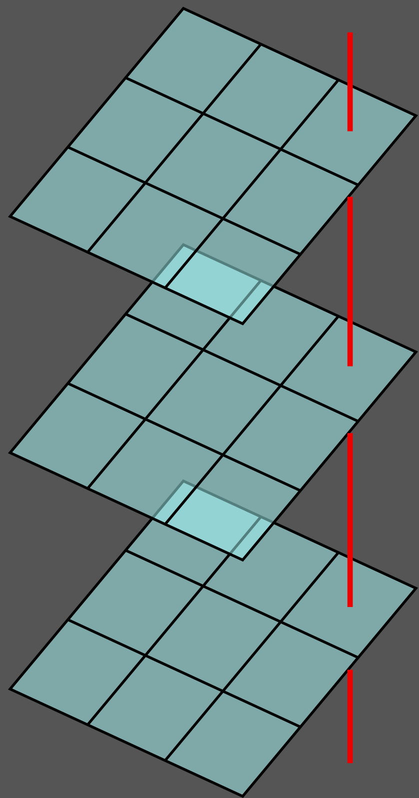
YOUR QUESTION

As one happy cyclist put it,'There's a good deal more going down than climbing in this race'. Exactly how many metres more going down is there than climbing ?

22 latin cube

You probably know what a Latin Square is : it's a square arrangement, 3 x 3 or perhaps 4 x 4 or 5 x 5, with numbers or letters or colours or shapes placed so that you never get the same thing twice in any row or column (though you are allowed the same thing repeated in a diagonal). Here's an example of a Latin Square :

1	2	3
2	3	1
3	1	2



Your problem is to place the numbers 1,2,3 on the three grids so that you never get the same number twice in the same row or column horizontally or vertically. For example, if you go down the red column you shouldn't meet the same number twice. . .

Think of the three square grids (pictured here on the left) as the three floors of a cubical building . . .

23 just two gorillas

In a Gorilla Rescue Centre in Eastern Congo (Africa), there's one enclosure with just two residents – a couple of young orphan gorillas called Sultan and Solomon. These two gorillas were rescued from poachers and they're now being cared for by a dedicated wildlife team. This team will also prepare the gorillas to go back to the jungle where they belong.

For the time being, the gorillas live on a special mix of food and sadly it's running out : there's just a certain number of sacks left and the team can't get hold of any more at present. So, that's a real problem for the team. Now here's your problem :

If they had only Sultan to feed, the remaining sacks would last him just 10 days. On the other hand, if they had only Solomon to feed, the remaining sacks would last him exactly 15 days. But of course both Sultan and Solomon have to be fed. Try to work out how long the remaining sacks will last . . .

Sultan

24 consecutive numbers . . .

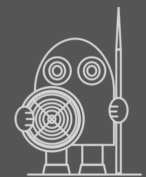
You know what *consecutive numbers* are – they're just numbers which follow each other. For example : 9, 10 are consecutive numbers and so are 11, 12, 13.

You can get various totals by adding together *pairs* of consecutive numbers; for example, 3+4=7 and 8+9=1 . . . Or you can add together sets of *three* consecutive numbers; for example, 5+6+7=18 and 9+10+11=30 . . . Or you can add together sets of *four* consecutive numbers, or sets of *five* consecutive numbers and so on.

a. You can get a total of 9 in two different ways : either from a *pair* of consecutive numbers (4 + 5) or from *three* consecutive numbers (2 + 3 + 4). Can you find another number under 20 where the same thing is possible?

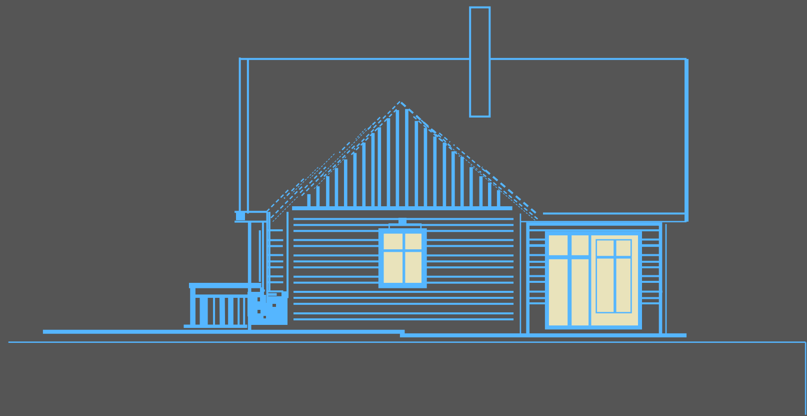
b. You can get a total of 18 in two different ways : either from *three* consecutive numbers (5 + 6 + 7) or from *four* consecutive numbers (3 + 4 + 5 + 6). Can you find another number under 40 where the same thing is possible?

c. Can you find a number under 50 which can be made either by adding *two* consecutive numbers or by adding *four* consecutive numbers?



25 the little house on the cliff . . .

John and his wife Amina used to own a small house on the edge of the cliffs somewhere near Newhaven. They bought the house for £100,000 but the following year there were newspaper reports that some cliffs in the area were unstable. The value of the little house fell by 20%. But then a new geological survey showed that their house was in fact built on solid ground. The value of the house went back up again to £100,000.



Exactly what was the percentage increase when the house's value returned to its original £100,000? (nb The answer is not 20%!)

prime numbers

26 sum of three primes

When Sally and her friends go into the maths room, Mr Pascal, the maths master, has taped a piece of paper onto the whiteboard. He tells the class that underneath the paper there are three prime numbers written on the board – and not just any three prime numbers chosen at random . . . these three prime numbers add up to 100.

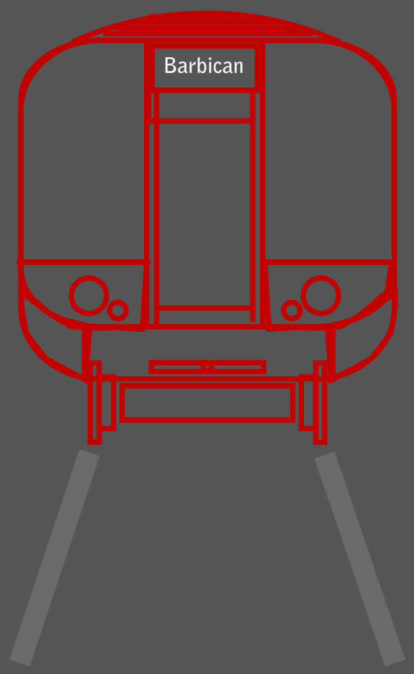
'What's the question?,' ask the pupils,'What is it you want us to work out?'

'The question is simply this,' says Mr Pascal,'What's the smallest of these three numbers?'

Remember : a prime number is a whole number with exactly 2 factors (itself and 1).

27 transport for London

120 people who work for Williams' Bank in the City of London were asked how they travelled to work. Altogether 82 reported that they travelled by train and altogether 45 reported that they travelled by tube. 10 said they didn't use either the train or the tube. How many used both the train and the tube to get to work?



28 marking time . . .

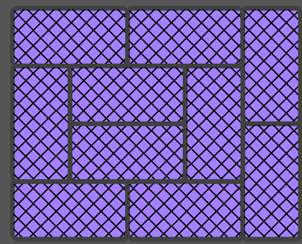
Here are Anthony's marks for the end-of-year exams. There were exams in six subjects – and those six teachers chose six different ways of marking, as you can see here. Because of this, Mr Barnes (Anthony's form teacher) is in trouble. He needs to work out an average mark for each boy and he just doesn't have the faintest idea of how to go about this.

Take a look at Anthony's marks and then see if you can find a sensible way of working out his average.

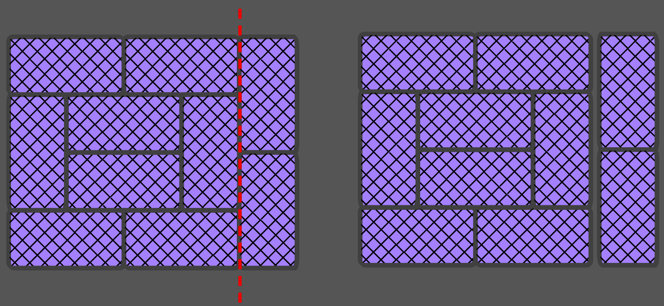
English	36 / 60
French	48 / 50
Maths	74%
Science	18 / 20
History	30 / 75
Geography	24 / 40

29 fault lines

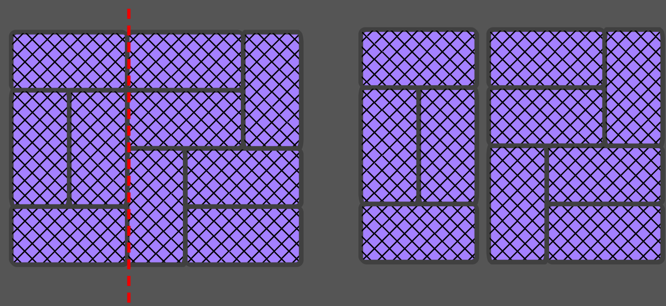
Suppose you have a number of 2 x 1 tiles. If you had a 5 x 4 rectangle, could you tile it completely with your 2 x 1 tiles? Well, the answer is yes – and in more ways than one. Here's one possible way :



But one thing we notice about this tiling is that it has a fault line, as you can see here :



And here's another example of a 5 x 4 rectangle tiling – and this also has a fault line :



** a 'fault line' is a line which runs right across the rectangle from one side to the other, in effect dividing the rectangle into two parts*

In fact, it’s impossible to tile a 5 x 4 rectangle with 2 x 1 tiles without getting a fault line somewhere. However it is possible to tile a 6 x 5 rectangle completely using 2 x 1 tiles and not to get a fault line.

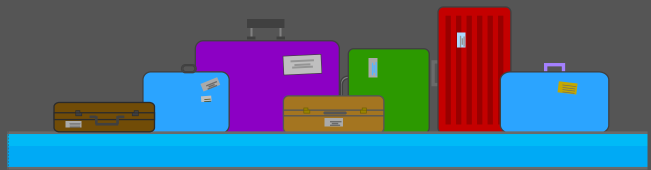
- Can you work out a way of doing it?

* *You’ll find dominoes, face down, make quite good plain 2 x 1 tiles if you're looking for a practical way of trying things out.*

30 an open and shut case

The Johnsons are going on holiday. Whenever they go away, the Johnsons always take lots of luggage with them. Mr Johnson is the worst culprit by far, as you can see from this information about their cases (to keep things shorter, we're calling them Father, Mother and Billy) :

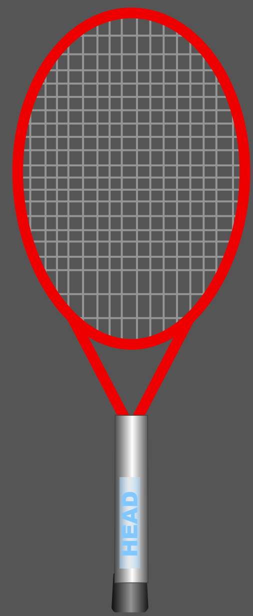
- Father's luggage weighs twice as much as Mother's luggage.
- Billy's luggage weighs only half as much as Mother's luggage.
- The difference in weight between Father's luggage and Billy's luggage is exactly 150 kg.



So, how much does Mother's luggage weigh?

31 what a racket!

Imagine you're organising a tennis tournament for your tennis club. Suppose by chance 16 people enter. You could start off with 8 matches, which would produce 8 winners. Then you could have 4 matches and so these 8 would be reduced to 4. Then 2 more matches (the 'semi-finals') would give you 2 winners. And these 2 could take part in the final match to produce 1 final winner, the 'champion'. It all works out easily, doesn't it?



Now suppose the following year only 14 players turn up. You would have liked 16 but you have only 14, so you give 2 players a 'bye' (that means they go straight through to the next round). Now you get the remaining 12 to play in 6 matches, giving you 6 winners. This means you have 8 players left (6 winners from matches they've played and 2 people who've had a 'bye'). With 8 players remaining, the rest is easy . . .

Now imagine the following year is a special Jubilee Year for your tennis club and you advertise the tournament in several places. To your surprise, 29 players enter. How many matches altogether will have to be played before you can end up with just one 'champion'?

32 late for work !

Mr Brodie was Chief Clerk in the Accounts Office of a small factory, where he was in charge of eight junior clerks. These clerks really did all the work (Mr Brodie just made sure they kept at it). The junior clerks were supposed to arrive at 7:30 each morning to start their day's work but in fact they turned up at all sorts of times. Mr Brodie was getting angrier and angrier about this until one morning he made a list of when each of the junior clerks arrived. This is the list :

Albert	*7:20*	*Ellie*	*7:36*
Ben	*7:35*	*Fred*	*7:27*
Carrie	*7:26*	*George*	*7:37*
Daisy	*7:22*	*Hattie*	*7:29*

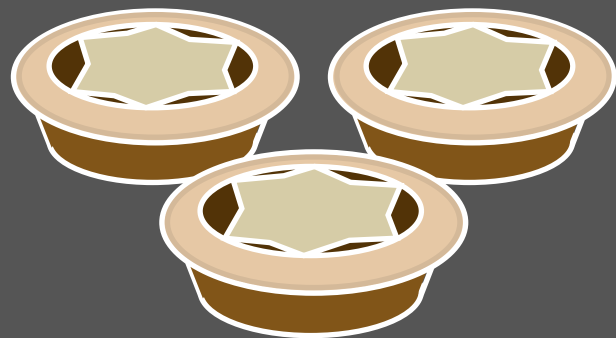
Mr Brodie looked at his list and straight away he began to shout at the clerks, saying were not being punctual . . . but then one of the clerks spoke up and said, 'Well, on average our times are not too bad!'

What was the **mean** (average) arrival time of the eight clerks?

33 tickets and pies

1 Last week, John went to a concert. On the way in, he bought a ticket for the concert and also a programme; altogether these two things cost him £11. In fact, the ticket cost £10 more than the programme. Don't do any working-out on paper – just think for a moment and then write down how much you think John paid for the programme and how much he paid for the ticket.

2 Sisters Sue and Jenny spent Sunday afternoon baking mince pies. When they had finished, there were 47 mince pies altogether. Some mince pies were for their own family and some were for the School Fair. Both girls agreed that the School should have 13 more mince pies than the family. So, how many mince pies did the family end up with?



34 a tricky question

It was 11:40 and Mr Pascal, the maths teacher, was sitting in the staff room at Low Moss School with a pile of books in front of him. He'd just finished marking the books and he was feeling rather pleased with himself. He looked up at the clock, saw that it was almost 11:40 and remembered that as it was the last lesson of the morning and as it was also a thursday, he should head for class 3N. He enjoyed teaching this class and they enjoyed his maths lessons.

As Mr Pascal entered the 3N classroom, he saw at a glance that the pupils were all there. Like many of the classes, 3N had an equal number of boys and girls. Mr Pascal asked the class a tricky question to start the lesson and within a minute or two, four girls had their hands up; the boys were still thinking and all had their hands down. At this moment there were one and a half times as many boys as girls with their hands down.

So, how many pupils altogether were there in 3N?

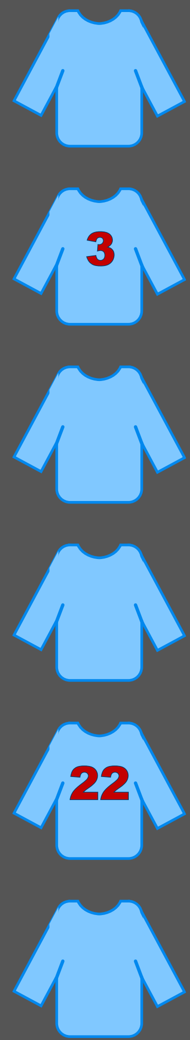
Form 3N

attention!

chiens méchants

35 6-a-side football

One Saturday morning, Jimmy and Hassan went down to their local gym for a six-a-side football game. Hassan's got a sister Amina who's a bit of a maths genius and when he arrived home, she asked him who had been playing for his team. 'Well, there were six of us down there', said Hassan, 'but I won't tell you their names. Instead I'll give you some info about their shirt-numbers!' First of all, Hassan said that obviously the six numbers were all different. Then, thinking of the numbers in a list, smallest first, he gave her these facts:

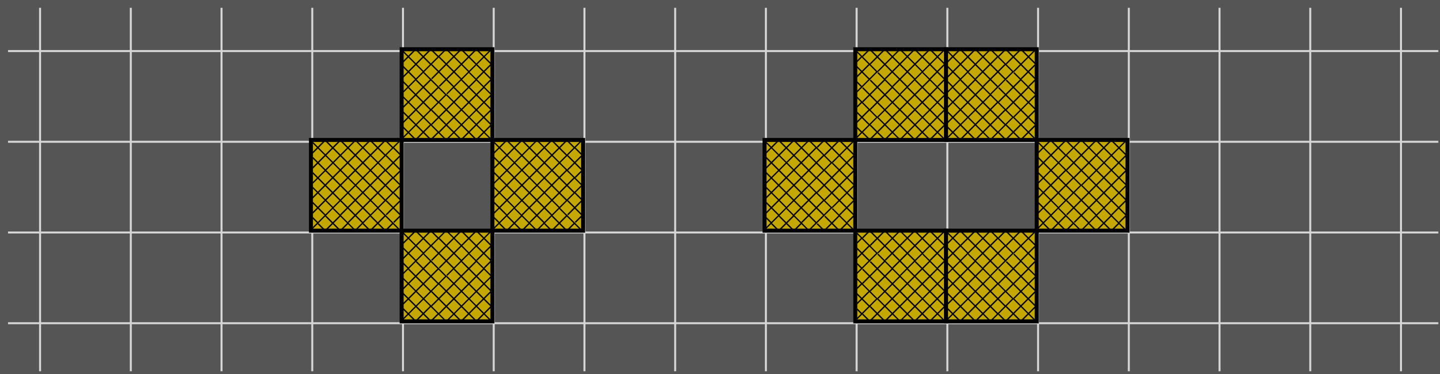


- The second number is 3 and the fifth number is 22
- If you add the first four numbers, you'll be just 1 short of the sixth
- If you square the second number, you'll get the third
- The fifth number is 5 less than the sixth
- The first three numbers add up to 13

What were the six numbers the boys' team had on their shirts ?

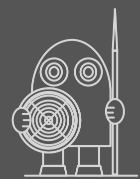
36 sheep may safely graze

On Tudor's farm, they use a 1 metre square grid (painted on the farmyard floor) and some bales of hay (they're one–metre cubes) to make sheep pens. When you're making one of these sheep pens, the bales of hay may just touch at the corners or they may lie exactly side by side – but there must be no gaps! Here are two examples of sheep pens from Tudor's farm :



As you can see, with 4 bales of hay you can enclose an area of 1 square metre – and with 6 bales you can enclose an area of 2 square metres.

What's the maximum area you can enclose if you've got 9 bales of hay?



37 Pablo's progress

PABLO PABLO PABLO PABLO

Summer-time! And once more the end-of-year exams are here. Pablo doesn't mind the maths exam because maths is something he enjoys. When all the exams are over and the marks come back, Pablo is pleased to find out that once again he's done fairly well on the maths exam.

For your information, this is how the maths paper is organised : the Mental Maths section is marked out of 20, the Calculations section is marked out of 30 and the Problems section is marked out of 50. So obviously if you have your actual marks for the three sections, you could just add them up and you'd get a mark out of 100, (which would, of course be your percentage).

But Pablo's teacher, Mr Ponticello, gives him his marks for the three sections like this :

Mental maths 85%   Calculations 90%   Problems 86%

Mr Ponticello then challenges Pablo to work out his overall percentage for the whole paper. Pablo is baffled – but not for long! Can you work out Pablo's final percentage ?

38 happy birthday James !

Happy Birthday! Happy Birthday!

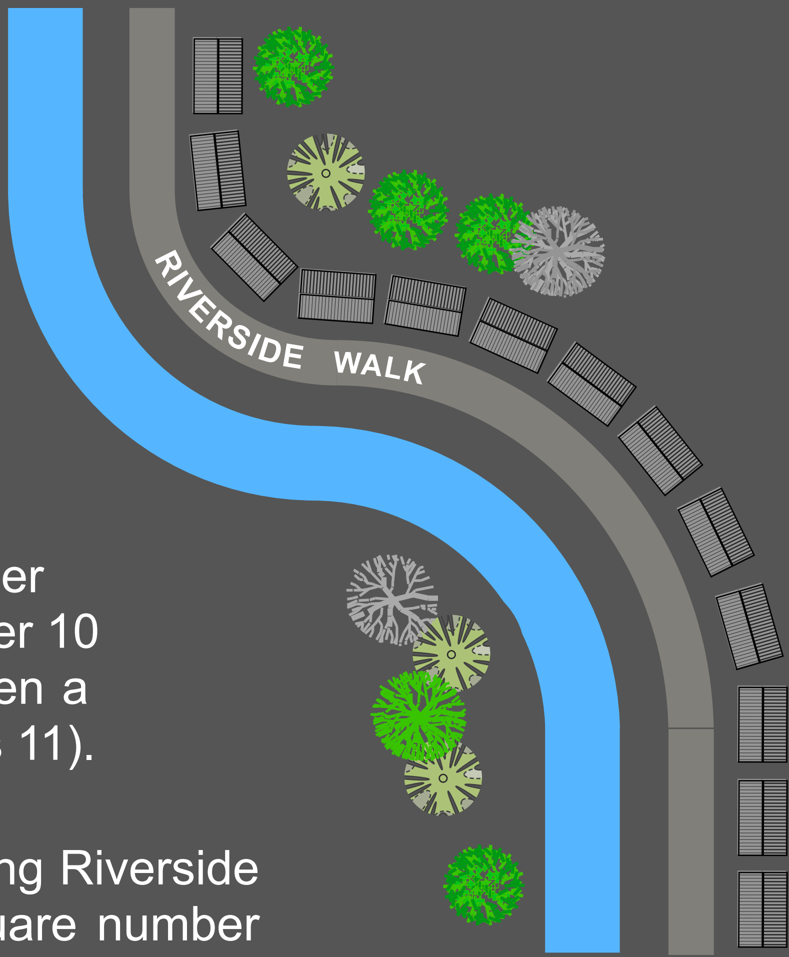
Sophie and her brother James have the same birthday. But they're not twins! Sophie is actually older than James; it just happens that their birthdays fall on exactly the same day. Here are two facts about their ages :

- Next year, Sophie's age will be three times James' age.

- Last year, Sophie's age was four times James' age.

From these two facts, work out how old James will be next year. Use any method you like to get an answer.

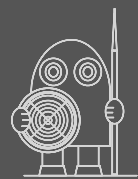
39 new year neighbours



Along Riverside Walk all the houses face the river and so they're numbered consecutively : 1, 2, 3, 4, 5 . . . and so on, up to 52. The Smart family move into number 10 Riverside Walk on 1st Jan. and young Alec points out that the house number in their address is rather special – they live at number 10 Riverside Walk – and 10 just happens to lie between a square number (that's 9) and a prime number (that's 11).

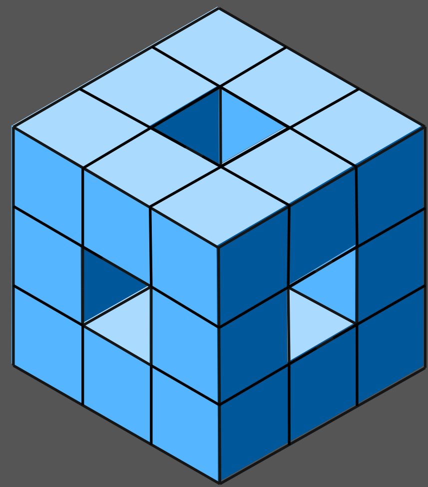
How many special addresses like this are there along Riverside Walk, with a prime number on one side and a square number on the other? Try to list them all.

optional extra : There's only one odd number among the answers. Why is this? (Hint : ask yourself whether there's anything different about the prime neighbour of the one odd answer.)



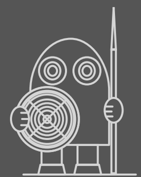
40 the heartless cube

Amy makes this shape by sticking together a number of 1cm wooden cubes. Her new shape is a 3cm x 3cm x 3cm cube – but with each face having a 1cm square hole going all the way through to the opposite face.



- How many 1cm cubes are there in the shape Amy has made?

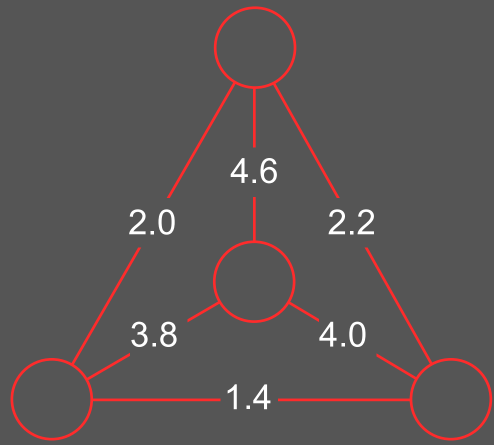
- Amy's shape is made of just plain wood but – she has some 1cm squares of sticky paper in all sorts of colours. She decides to cover every part of her shape (that's to say, every part which is open to the air), using different kinds of blue. How many of these 1cm sticky squares will Amy use? (Or, to put the question a different way, what's the total surface area of Amy's shape?)

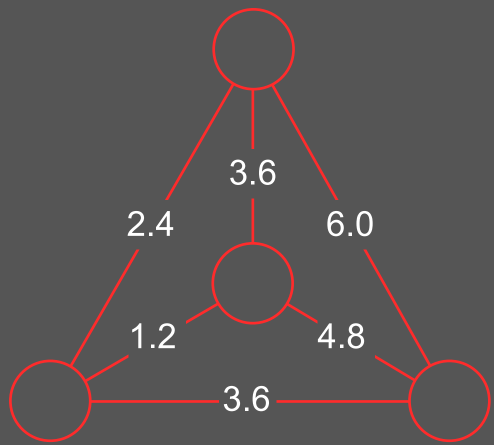


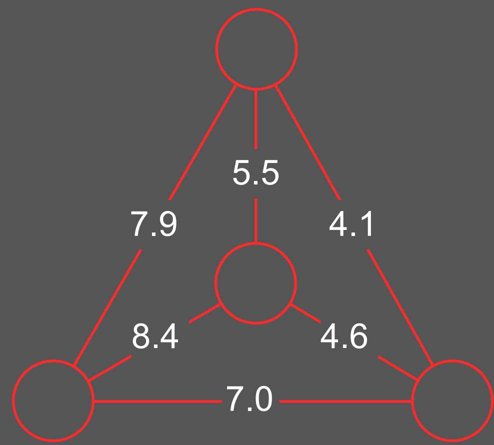
41 more number triangles

Now number triangles just got a whole lot harder ! You've come across these before and you probably remember how a number triangle works : pick any two circles and the number between them is what the numbers in the two circles add up to. However – we've blanked out the numbers in the circles and you have to figure out what the originals must have been.

You've done this before with easier numbers and perhaps then you found a way of getting started and of working through to an answer. That's what you need to do in order to solve these three problems.



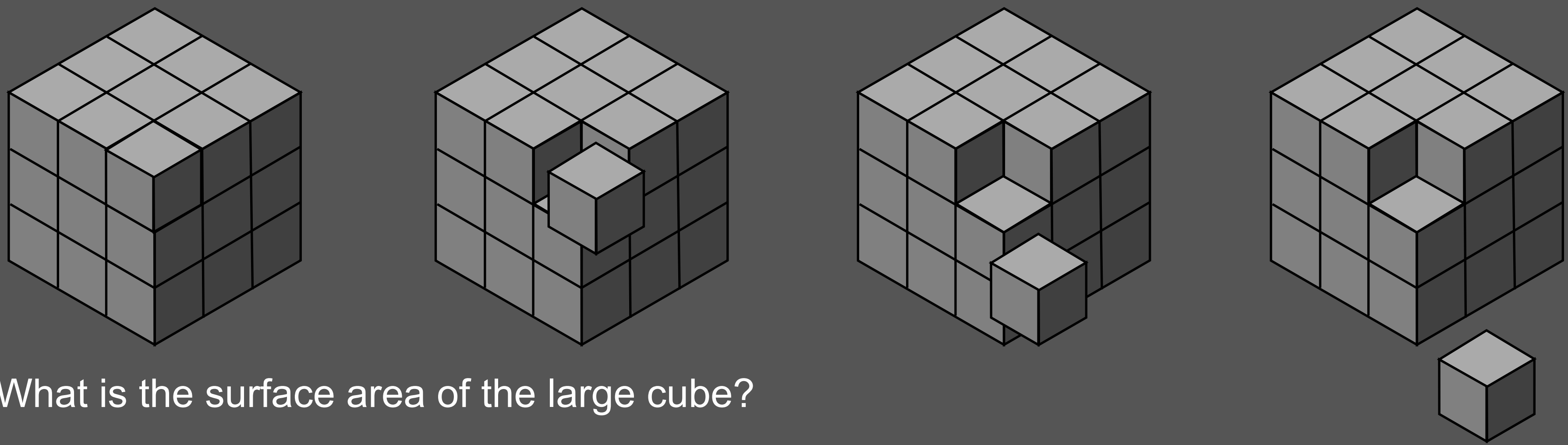




42 the missing cube

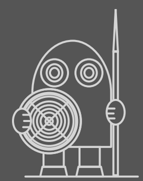
The large cube below (left) is made up of smaller 1cm cubes. You probably know what we mean by the *volume* of this large cube : it's just how many 1cm cubes it contains (27 here).

Surface area is a different idea : To work out a shape's surface area, you just find the area of each face of the shape – and then add all these areas together. As a cube has six identical faces, there will be six areas (all the same) to be added together. Here are two surface area problems for you to work out :



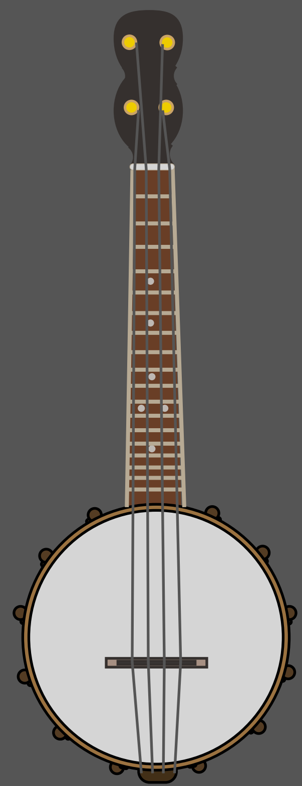
- What is the surface area of the large cube?

- The large cube has not been glued together very well and one afternoon one of the small corner cubes comes loose and eventually falls off (see pictures 2, 3 and 4 above). So now we have a large cube with one missing corner, together with a small cube. What's the total surface area of the large cube now it has a corner missing?



43 the frog and banjo

A few years ago there was a very successful pop group called '*The Amphibious String Band*' (you know what *amphibious* means). The group had a frog, two newts (called Newt 1 and Newt 2) and a toad; they mostly played country-and-western music. Because the frog was really the star of the group, they later changed their name to '*The Frog and Banjo*'. Each group member played only one instrument; here's some information about who played what :



- Newt 1 did not play double-bass
- Toad didn't play either lead guitar or rhythm guitar
- Frog played the banjo (pictured here)
- Newt 1 did not play rhythm guitar
- Newt 2 didn't play lead guitar

Can you work out who played which instrument?

44 cube calendar - months

You might remember book 1 problem 42 'calendar cube - days'. That problem was about a daily calendar made of wooden cubes. The numerals on the first two cubes showed the day of the month – and the letters on the remaining three cubes told you which month it was. The calendar was made by Mr Pascal, a maths teacher, using five plain wooden cubes and some white stick-on pvc numerals and letters.

In book 1 problem 42 you had to find a way of sticking numerals onto the first two cubes so that all dates in the month (that's to say, all numbers from 1 to 31) could be shown. The problem here is a rather harder one. Your challenge now is to think of a way of sticking letters onto three cubes so that by lining up the cubes in different ways you can show any of the twelve months from 'jan' to 'dec'. There are various ways of going about this but it's not an easy problem.

hint : If you need to, you're allowed to use the 'n' to also stand for 'u' – and you're allowed to use the 'd' to also stand for 'p'. (With both the 'n' and the 'd', turning the cube upside-down will allow the one letter to look like the other. And you'll find that in most sets of self-adhesive numerals you can buy, the 'n' and the 'u' are exactly the same shape as each other, as are the 'd' and the 'p'.)

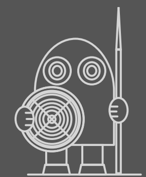
45 stairway to heaven

Soldier ants guard their colony and as you might guess, they like to keep fit. Milit is an ant just like this. To keep himself in trim he sets off every morning at the bottom of the church tower and he makes his way to the top by starting on step 1 and then jumping to step 4, then to step 7, then to step 10, then to step 13 and so on . . .

One morning, just as Milit starts to jump from step 1, a rather fat frog jumps from the top step (step 161); the frog then jumps onto step 157, then onto step 153, then onto step 149 and so on . . .

Both the frog and the ant jump exactly in time with the regular chiming of the church clock. This means that you have ant on step 1 at the same time as frog on step 161, then ant landing on step 4 just as frog is landing on step 157 and so on . . .

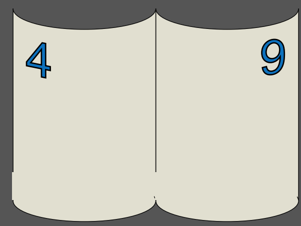
If the frog lands on a step at the same time as Milit the ant, then Milit will probably get squashed and die. Try to work out whether this terrible thing will really happen.



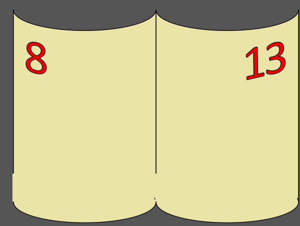
46 after the flood

It's that time of year and once again floods have hit the little Cornish port of Fowey. Waters swept through the printer's shop and many of the leaflets he had prepared for his customers simply floated away.

Jenny found one loose sheet from a booklet floating in her garden; she wasn't sure what the booklet was about but the page was A4 size with a fold in the centre – clearly it had originally been part of an A5 booklet. The loose page she had was numbered 4 and 9 on the one side, and 3 and 10 on the other side. How many pages were there in the original booklet ? And how many single A4 sheets had been used to make the booklet ?



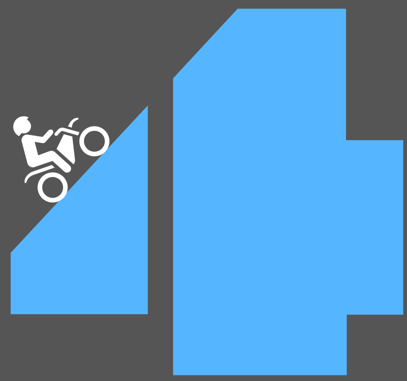
Roger came across a single sheet from a completely different booklet. Again, the page had been part of a small A5 booklet. The page-numbers on one side of the sheet were 8 and 13 and on the other side of the sheet the page-numbers were 7 and 14. How many pages were there this time in the original booklet? And how many single A4 sheets had been used in making the booklet ?



47 action fractions

Here's an unusual problem involving fractions :

What what number can you subtract from 4 and get the same result as when you multiply it by 4 ?



This number, as you might guess, is a fraction and because it's a fraction which can do two things, we call it an 'action fraction' : take it away from 4 and you get a certain answer; or multiply it by 4 and you get exactly the same answer. See whether you can find the action fraction we're looking for here.

USEFUL HINT : the action fracton we're after here is less than 1 . . .

48 trip saver

Every year at the start of October, Meriden High School runs a trip to London for 6th-form pupils. As well as various visits to well-known London places, the trip involves a 1-night stay in a Kensington Hotel. As with everything in life, there's a price to pay; in this case the overall cost of the trip is £80 per pupil. Both Khori and his sister Liss want to go on the London trip; but for each of them this will have to involve some careful saving-up. Here's how it goes :

LONDON BRIDGE
TRAFALGAR SQUARE
MAIDA VALE ABBEY ROAD
ROYAL ALBERT HALL
KNIGHTSBRIDGE
WATERLOO KINGS CROSS
MARBLE ARCH SOHO
ST. PAUL'S CATHEDRAL
PICCADILLY CIRCUS
WESTMINSTER ABBEY
OXFORD CIRCUS

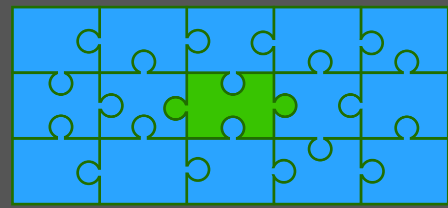
Liss says there's no time like the present and so she begins to save in the first week of the year; let's call it week 1. Liss handles her spending-money carefully and she saves £2 each week without fail. Khori says he needs all his spending-money, so he doesn't start saving for the trip until he's found a part-time job delivering leaflets. Each week then, starting in week 9, Khori does his few hours of paid work and each week from week 9 onwards he saves £3.00 towards the trip. He's started later than his sister but he's saving faster!

By the way, all money for the trip must be paid for by the end of week 40.

One week Khori and Liss find that they have paid exactly the same amount.
(a) In which week of the year does this happen? (b) What is the amount?

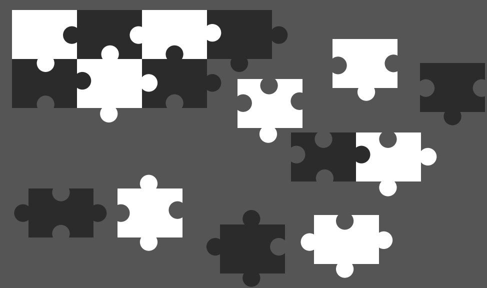
49 black & white jigsaw . . .

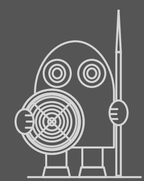
Here's a picture of a very simple jigsaw (to make it easier to picture, we've made the main inner edges straight). As you can see, there's one piece, coloured green here, which you could call the 'central piece' – meaning that it has the same number of pieces above and below it **and** the same number of pieces to the left and to the right.



Perhaps you remember an earlier problem about Syed and his jigsaw of 1458 pieces. That jigsaw had 54 pieces along each longer side and 27 pieces along each shorter side. Did Syed's jigsaw have a 'central piece'?

Next, once again thinking of Syed's jigsaw and its 1458 pieces : Suppose the pieces of this jigsaw were coloured black and white in such a way that you could lay out all the pieces and never have a black edge next to a black edge or a white edge next to a white edge. Would there be exactly the same number of black and white pieces?





50 match days count

You know how it is with a football league - there are days when all the clubs play and there are days when only some of the clubs play (and the others have a rest day). Of course, how easy (or how hard) it is for the organisers to work out a programme of match-days depends on how many teams there are in the league.

In Hythe there are just 4 teams who compete every Autumn half-term for the Hythe Schools Cup. The competition is run as a league – that's to say, every school must play every other school and points are awarded for either a win or a draw. All the matches take place in the Hythe Town Stadium, so there's no need for 'home' and 'away' matches. And of course, no team is ever expected to play twice on any day.

As you can probably see, 6 matches altogether must be played to complete the competition. It's easy for the organisers to fit these 6 matches into 3 match days (with no need for any rest-days!). The chart on the right shows one way of arranging things; as you can see, it shows team A playing team B and also team C playing team D on the first day – then on the second day, team A plays team D and team B plays team C – and on the third day, teams B and D play one match whilst teams A and C play another.

4 teams

	resting
A B / C D	–
A D / B C	–
B D / A C	–

= *3 match-days*

Problem : What's the smallest number of match-days you would need if another school joined in the competition (making 5 schools in all) ?

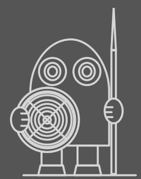
mark time . . .

This looks like the sort of problem where *experiment* might well be the best approach. Let's try some different ages for Mark and see what happens :

Susan	Mark	Susan	Mark	Susan	Mark
3	1	6	2	9	3
4	2	7	3	10	4
5	3	8	4	11	5
6	4	9	5	12	6
7	5	10	6	13	7
8	6	11	7	14	8

To begin with, we've just worked out what happens when Mark is 1 or 2 or 3. As you've probably worked out by looking at our lists, the two red lines link the children's ages now and in two years time.

It doesn't take long to see that if we have Mark as 2 years old now, then everything works out fine. Reading from the middle set of results, we can see that our answer is : in two years' time, Susan will be 8 and Mark will be 4.



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ps. You might have realised at the beginning that Mark's age now must be an even number. This makes life a lot easier, as it halves the number of ages we need to try out for Mark. Why must Mark's age today be an even number? Here's the thinking behind the idea . . .

1 In two years' time, Susan's age will be exactly double Mark's age . . .

2 Which means that in two years' time, Susan's age will be an even number . . .

3 So Susan's age must be an even number now (now differs from then by exactly two years) . . .

4 And so Mark's age must be an even number now (because if it were odd, then three times it would give an odd number for Susan's age) . . .

2 shopping day

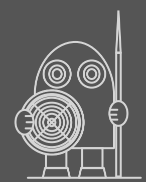
One way of setting about this problem is to draw up a grid, like the one on the right, showing the days of the week in one direction and the names of the shoppers in the other direction :

	M	T	W	Th	F	Sa	Su
Sam							
Lucy							

Next, we can add symbols showing what we're told about the weather on different days :

	M	T	W	Th	F	Sa	Su
Sam							
Lucy							

PTO



ans 2 shopping day

Now we look at the facts we're given about Sam and for each thing we're told, we put a cross in every square where we know he can't have been shopping. (For example, fact 1 tells us that, 'It was raining when Sam went shopping'. So we can put a cross for him under all the dry days.) Then we look at the facts for Lucy and put crosses wherever she can't have been shopping.

	M	T	W	Th	F	Sa	Su
Sam				X	X	X	X
Lucy	X	X	X			X	X

Taking a hard look at the table we've drawn up, we can see the answer to our problem. We're told that Lucy went shopping the day after Sam and, as you can see, the only days when this happened were Wednesday and Thursday. So there we have it :
Lucy went shopping on Thursday; Sam went shopping on Wednesday.

	M	T	W	Th	F	Sa	Su
Sam			✓	X	X	X	X
Lucy	X	X	X	✓		X	X

ans 3 mean cyclists

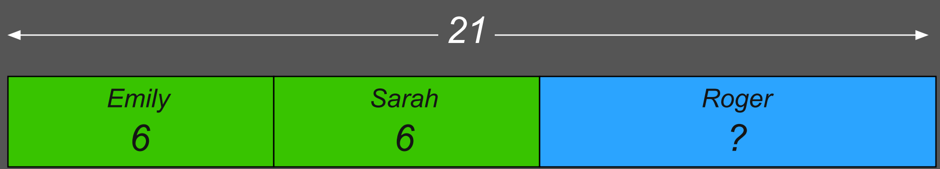
This question is about means, so stop for a minute and think about how you work out means : you add up a set of numbers and then you divide by how many numbers there are. That's easy and you've probably done it lots of times. If someone tells you that five numbers add up to 20, you can easily work out the mean of the numbers : it's just 20 divided by 5, that's to say 4.

Now imagine someone tells you that they're thinking of a different set of numbers; this time there are eight numbers and their mean is 6. Can you work out what the numbers must add up to? If you're stuck, just stop and think how the mean was worked out in the first place : the total was divided by 8. The only number you can divide by 8 and end up with 6 is of course 48. So the total of these eight numbers must be 48.

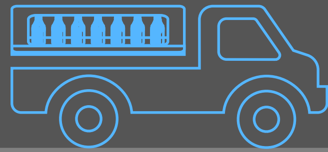
Let's take another look at our question. We have three numbers (ages in this case) and their mean is 7. What must their total be? Answer : 21

This total of 21 is made up of two lots of 6 (the twins' ages) plus another number (Roger's age). This other number has to be 9 (that's 21 – 12). So Roger is 9.

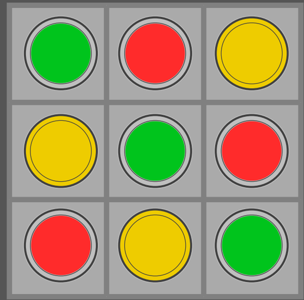
Here's a diagram to illustrate :



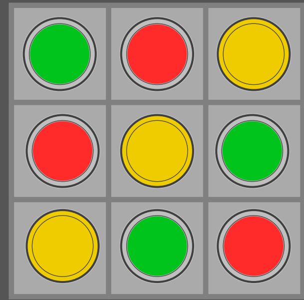
ans 4 **bottle-tops are go!**



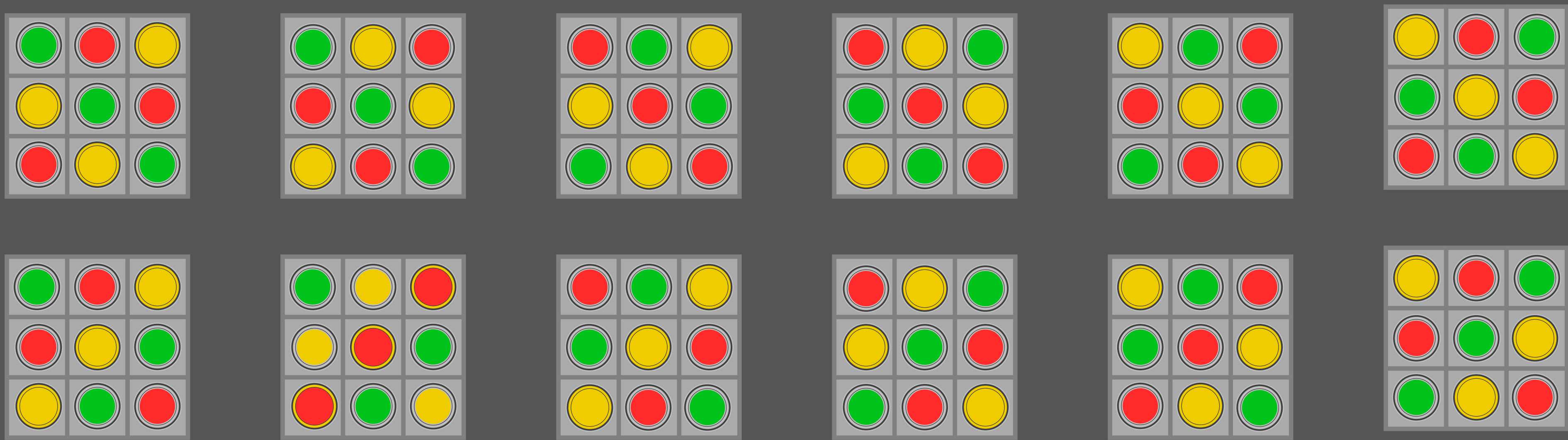
There are really just two different ways of producing a 3 x 3 Latin Square. You can start with your first row and then 'move it along one place' for the next row and then 'move it along another place' for the last row :

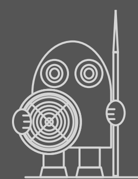


. . . or you can take the same first row and then 'move it along 2 places' for the next row and then 'move it along 2 more places' for the last row :



. . . with these two different ways of producing a Latin Square and six different first rows, not surprisingly you will end up with 12 different Latin Squares. How many of these did you find?

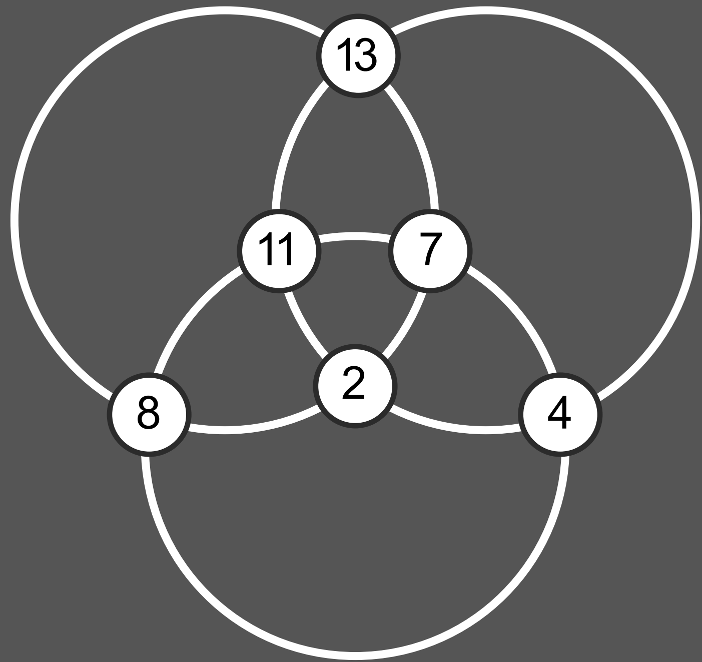




ans 5 Japanese Magic Circles

There's no obvious way to solve this problem – other than just trying the numbers in different places until you get a feel for how things work. Sooner or later you'll find a successful arrangement. On the right is one answer : if you check it, you'll find that the totals around the three circles are all the same. However . . .

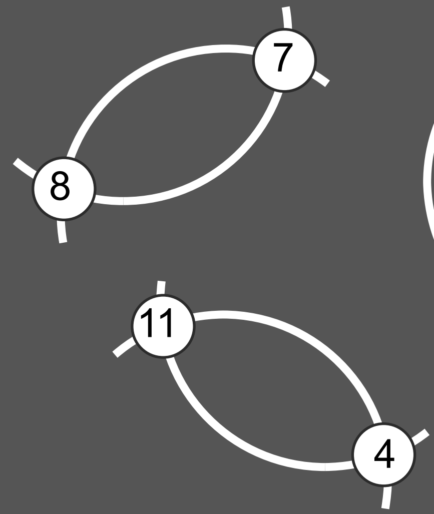
. . . you might have found a different answer – and yes, this is one of those problems where there are a number of different answers, all of them correct.

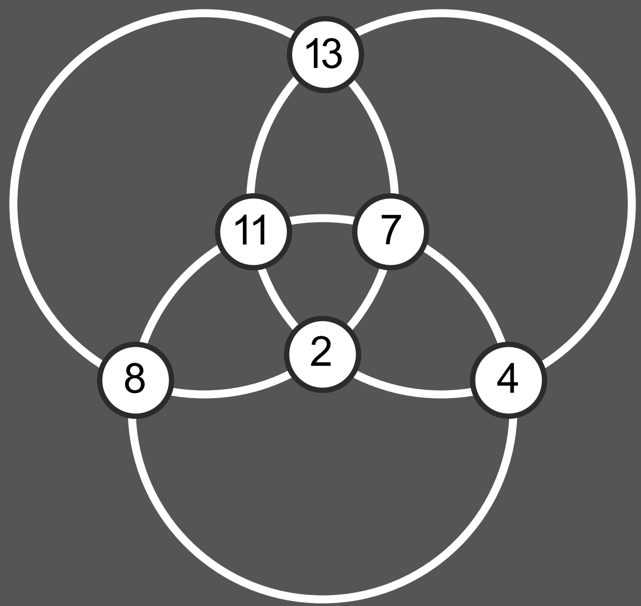


PTO

ans 5 Japanese Magic Circles

There's an interesting pattern to be found in these Japanese Magic Circles : Starting with the solution given on the previous page, just look at each of the 'rugby ball' shapes in the diagram and add up the numbers at opposite ends, you'll see that you always get the same total, that's to say, 15 !

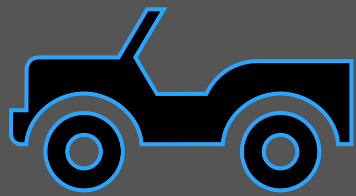




There are many ways of coming to an answer : but it turns out that every correct answer to the problem has these same three pairs of numbers sitting opposite each other. That's to say, if you've got three 'rugby balls', one with 13 opposite 2, one with 11 opposite 4 and one with 8 opposite 7 – then you've got a correct solution !

Perhaps you can see how this simple fact can be used to get you to a correct answer . . .

ANS 6 the job's yours !



This is a fairly easy logic problem – as long as you keep a clear head about what's required . . .

There are just three things which the new team-member really has to be : **1** good driver, **2** gun-handler, **3** master of disguise. Everything else we can ignore. Now let's take these three things in turn :

good driver : Pete is ruled out as he can't drive

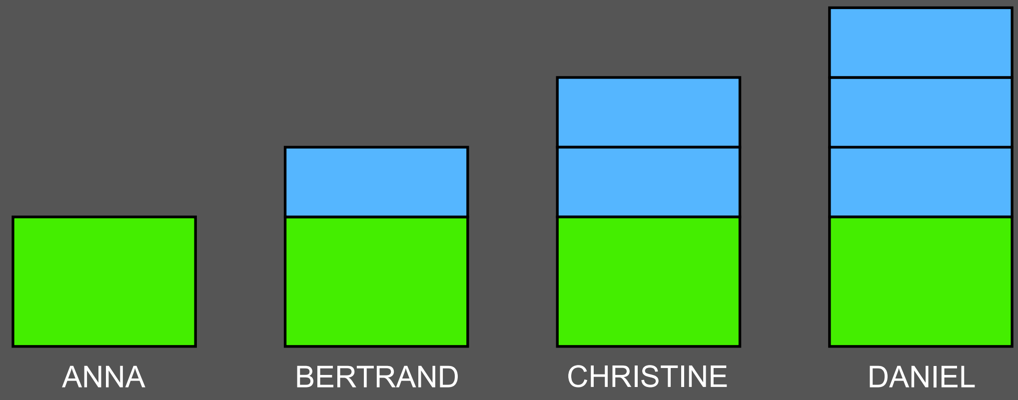
gun-handler : Frankie is ruled out as he can't handle guns

master of disguise : Jake is ruled out as he isn't good at disguise

. . . and that leaves Sam. Which means that Sam will be joining The Major's team.

ANS 7 **spaced-out kids**

Let's see if there's a diagram which could help us with this problem . . .



We don't know what Anna's age is, so let's just show it by a simple green box.

We do know that Bertrand is 2 years older than Anna, so we can show his age by the same green box plus something to stand for the extra 2 years he has above Anna : we've used a blue box to stand for 2 years

Christine is 2 years older than Bertrand, so we can show her age as the same as Bertrand's, plus another blue box

And of course Daniel is 2 years older than Christine, so we've given him yet another blue box.

PTO

spaced-out kids

Where do we go from here? Well, we do know that all the ages together add up to 28. Looking at the diagram, this must mean that :

4 green boxes plus 6 blue boxes makes 28

But the blue boxes are each worth 2, so six of them must equal 12 . This means that:

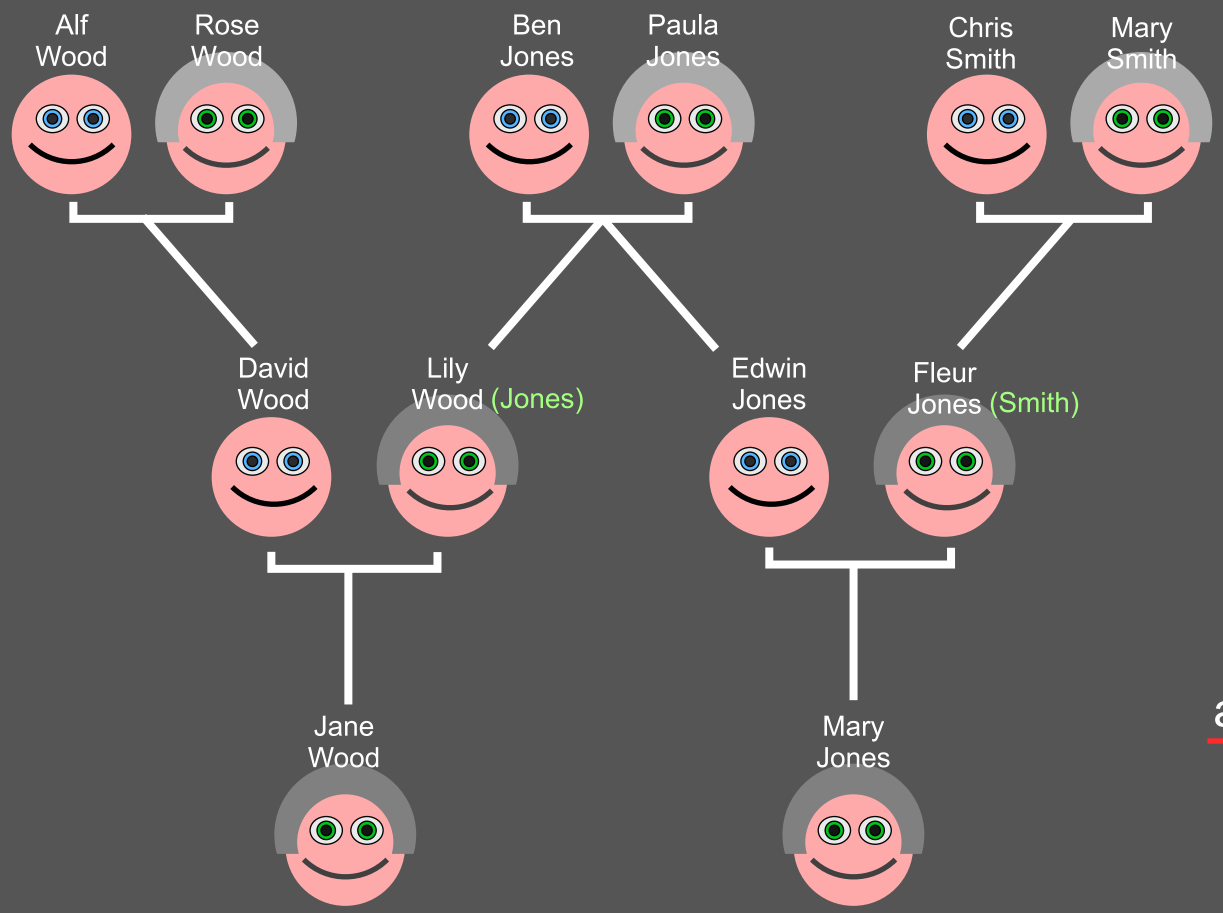
4 green boxes plus 12 equals 28

– which means that the 4 green boxes must add up to 16. In other words, each green box must be worth 4. From this we can easily work out that :

Anna must be 4 and so Bertrand must be 6.

ans 8 **Jane's people**

If Jane and Mary are cousins, then one of Jane's parents must be brother or sister to one of Mary's parents. So of course, they'll have just two grandparents in common. Here's one way in which it could happen in real life :

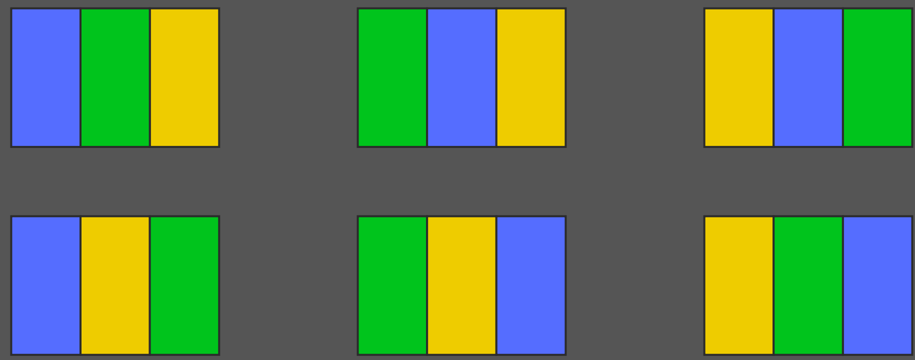


* the names in brackets are the names they were born with

answer : 2

ans 9 flag day

- In all, six different arrangements are possible. Here they are :



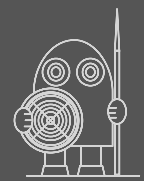
Without making a list or drawing and colouring a full set of flags, you can get to the same answer by this reasoning :

STRIPE 1		STRIPE 2		STRIPE 3		
3 choices	x	2 choices	x	1 choice	=	6 arrangements

- There are six possible arrangements and, as you can see, just two of them have blue between two other colours. So, we can write :

probability of blue between two others = 2/6 = 1/3

special note : *in maths, we always show probabilities as fractions*



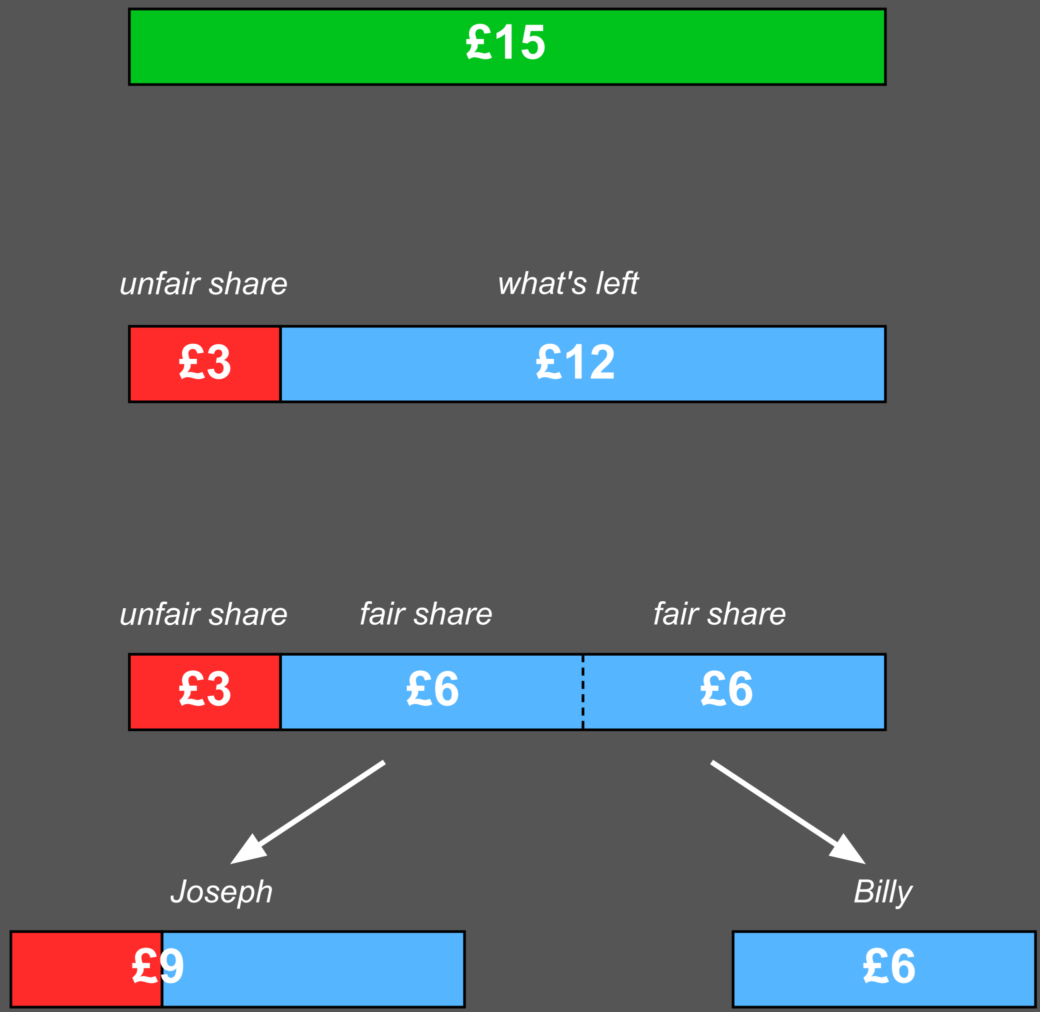
ans 10 it's a clean sweep!

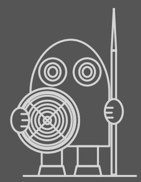
Remember, diagrams are often useful to show what's going on in a problem. Let's start off by drawing a single bar to stand for the £15 which the boys earn :

Now lets split this bar into two parts, a £3 and a £12. The £3 is the 'unfair share' (at least that's what Billy calls it) and this is what Joseph gets. The £12 is what's left after you've removed the 'unfair share'.

If we now take the £12 and divide it equally into two parts (each £6), we can call these parts the 'fair shares'.

Finally, we give the money to the boys: £9 to Joseph (you can see why it's £9) and just £6 to Billy. 'Life is never fair', says Billy.





PTO

ans 10 it's a clean sweep!

Some people prefer to set this kind of problem out in a different way – without the need to draw diagrams. So, here's one way of doing that. The most important thing to remember though is that it's usually better to deal with the 'unfair share' first.

Let's keep what the two boys are getting in two separate columns. Also,in maths we like to shorten things, so let's call the fair share the F.sh and the unfair share the Unf.sh . . .

	Billy	Jos.
Unf.sh		3
F.sh		

First we write the £3 Unf.sh under Jos.

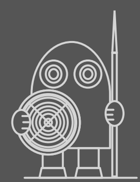
	Billy	Jos.
Unf.sh		3
F.sh	6	6

Next, we divide the £12 we have left over into two equal fair shares of £6 and put these as F.sh

	Billy	Jos.
F.sh		3
Unf.sh	6	6
	6	9

Finally, we add up the columns to get a total for each boy.

Well, not quite finally: we need to check that our answers match what the question is asking!



ans 10 it's a clean sweep!

So far we've looked at two different ways of tackling this problem, that's to say using diagrams and setting things out in a table. Of course the numbers in this problem are pretty small, so you might well think that we've made the whole thing unnecessarily complicated. It wouldn't take you long just to try a few numbers until you find an answer which works . . .

. . . for example, suppose we guess that Billy's share is £4. Then we know Joseph gets £3 more than Billy, so Joseph must get £7. But these two amounts, £4 and £7, don't add up to £15; in fact, they add up to just £11. So we need to try again, this time by guessing a larger share for Billy . . .

. . . So, this time let's guess that Billy's share is £5. Then Joseph's share must be £3 more than this, that's to say, £8. Now the two shares add up to £13, so we're getting nearer. Perhaps a guess of £6 for Billy's share will do the trick . . .

Well, this wasn't too hard at all. But you'll find you often come across problems just like this one (we call them 'unfair sharing' problems), except with harder numbers – and then one of the other methods we've shown you might suit you better.

heads and tails

Let's start by taking a careful look at what can happen when you toss two ordinary coins into the air. Perhaps your first reaction is to say that there are only three results possible : two heads, two tails or a head and a tail. That's good as a general way of describing things – but when it comes to calculating probabilities . . .

. . . we need to be very precise. Let's call our two coins SILVER COIN and BRONZE COIN. Now we can see that there are the four equally likely results :

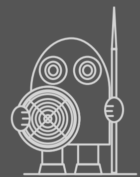
SILVER COIN	H	H	T	T
BRONZE COIN	H	T	H	T

- The first question asks us to work out the probability of getting two heads. We can see straight away that this happens just once out of the four possible results. So, prob (two heads) = 1/4

- The second question asks us to work out the probability of getting a head and a tail. We can see that this happens twice out of the four possible results. So, prob (a head & a tail) = 2/4 = 1/2 ans 12 **will there be time ?**

Here are all the palindromes the digital clock will show between midnight one night and midnight the following night. Altogether there are 16 different ones . . .

00:00	10:01	20:02
01:10	11:11	21:12
02:20	12:21	22:22
03:30	13:31	23:32
04:40	14:41	
05:50	15:51	



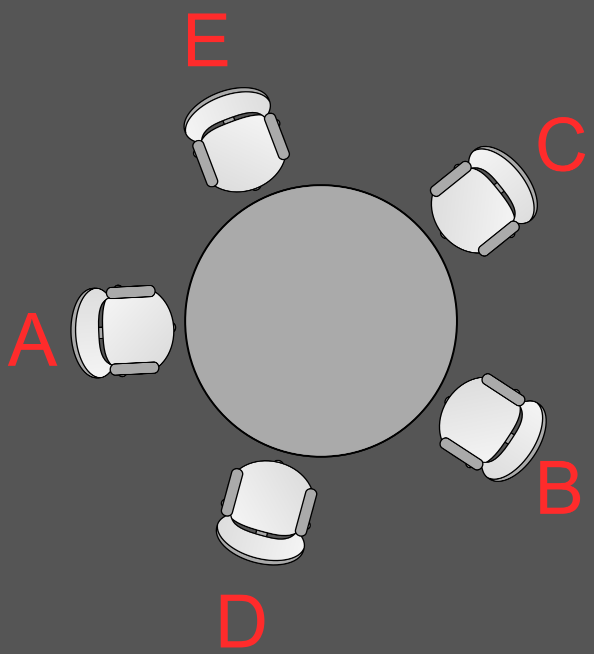
ans 13 mean Mr Francois

First of all, remind yourself of how we get mean averages in the first place : we add up a set of figures and then divide this total by the number of figures we've added. For example, suppose you want to average these four numbers : 3, 7, 10 and 12. Adding these numbers together gives a total of 32. There are 4 numbers here, so we divide our total by 4; and since 32 ÷ 4 = 8, our (mean) average = 8.

In the Mr Francois problem, we can't begin by just adding up the numbers, as we don't have them all. But we do know that there were five numbers – and we do know that their average = 7. So the total must have been 35 (since 35 ÷ 5 = 7). Now we're almost there! The four numbers we are given add up to 25, so the missing number must = 10. Friday must have been a very bad day indeed for Mr Francois (and for his pupils).

answer : on Friday Mr Francois got angry 10 times.

ans 14 come round for a meal !



On the left is one way to arrange the seating so that everything works as it should. Your diagram might look a little different from this but if your letters are in the same order as you go round the table, then it's a good answer. For example, starting at A and going clockwise, you should get the order :

A E C B D

Answers to the questions :

1 Charles is sitting on Ellie's left

2 Beatrice is sitting on Diana's right

- *with numbers larger than 400 but smaller than 500, the smallest digit-sum is 5 :*

coming
from
either 401
or 410

- *with numbers larger than 400 but smaller than 500, there are nine ways of getting a digit-sum = 12 :*

408
417
426
435
444
453
462
471
480

- *with numbers larger than 400 but smaller than 500, the largest digit-sum is 22 :*

coming
from 499

ans 16

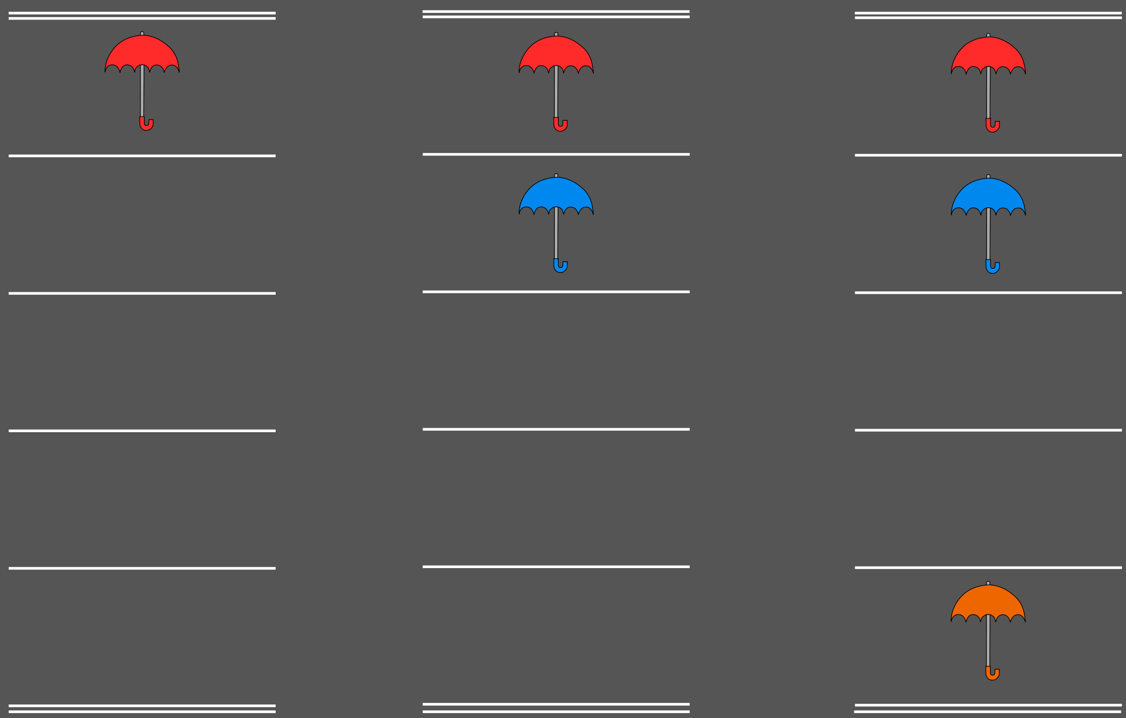
singin' in the rain

One way to tackle this problem is to make a chart with five spaces (there are six umbrellas but remember, two of them share a place . . . and then to go through what we're given and gradually fill in the chart :

we know that red is first on the list . . .

. . . and we know that blue is second

there were three colours between orange and blue – so orange must be last

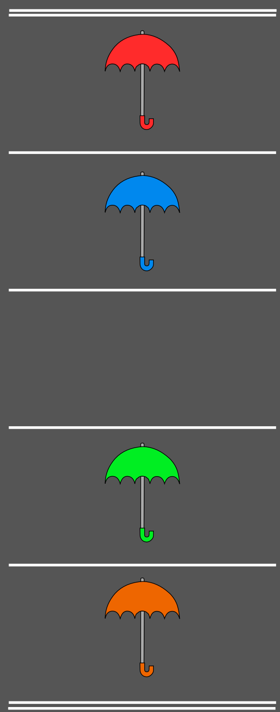


PTO

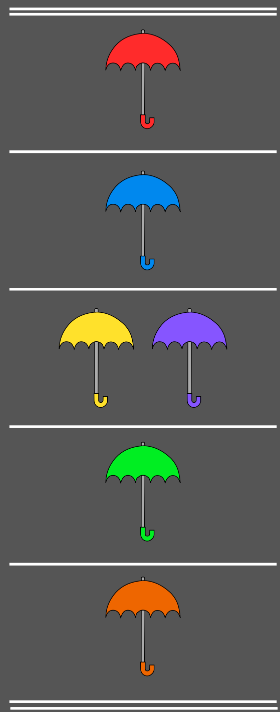
ans 16

singin' in the rain

there were two colours between blue and green – so green must be just above orange



this leaves the last place to yellow and purple (we know they are equal)



– and this final column gives us our answer : red 1st, blue 2nd, yellow and purple 3rd place equal, green 5th and finally, orange 6th ans 17 **no red faces here !**

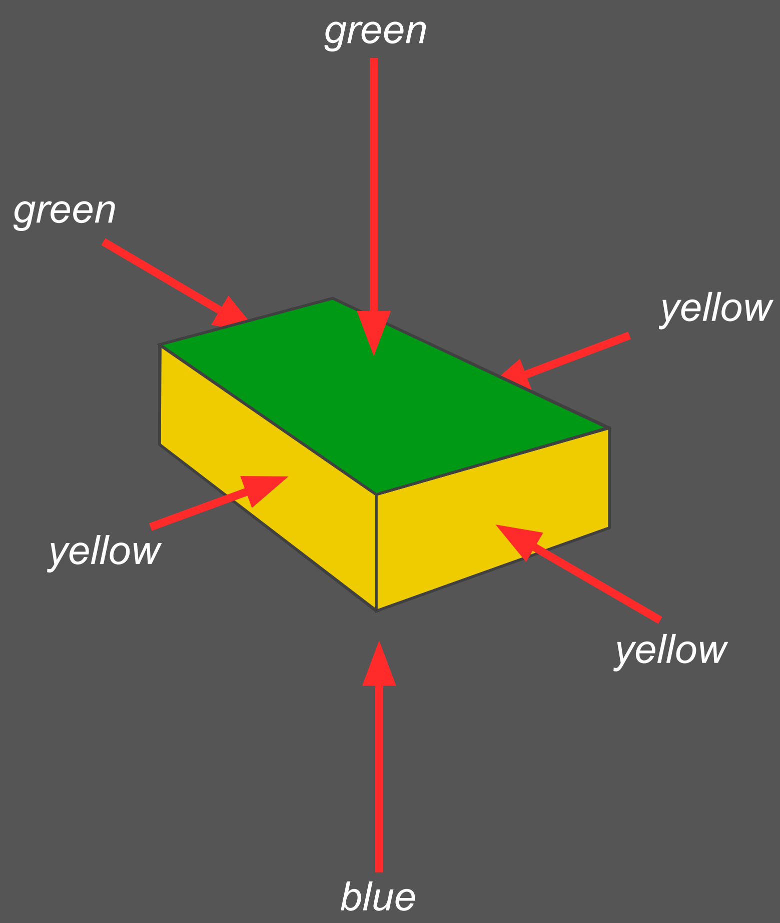
there are two green faces (fact 1), so no long side faces are green and only one end face is green

the top and bottom faces are green and blue

fact 2 is nothing new – it's already covered by fact 1 and what we can see

fact 3 tells us that no long face is blue, so now we can be sure that the end face we can't see and the long face we can't see must both be yellow

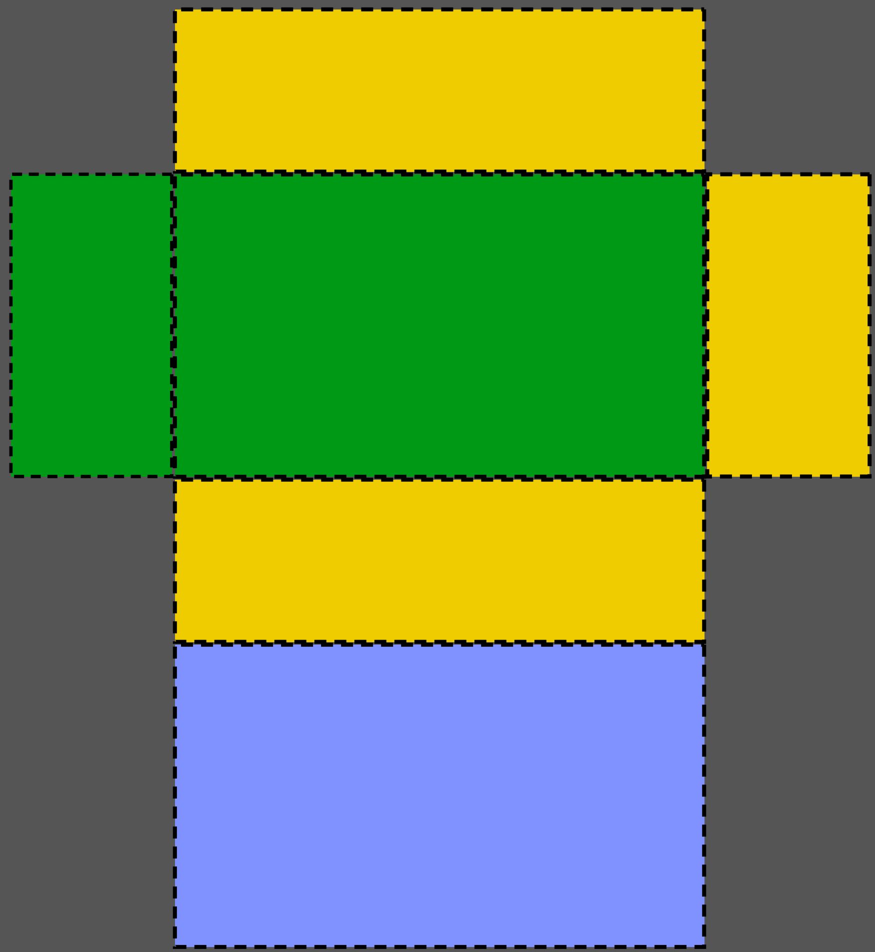
– so altogether there must be three yellow faces (that's two long faces and one end face)



PTO

ans 17 **no red faces here !**

For anyone who might like a 2D version of the answer, here's a net for making this colourful box :

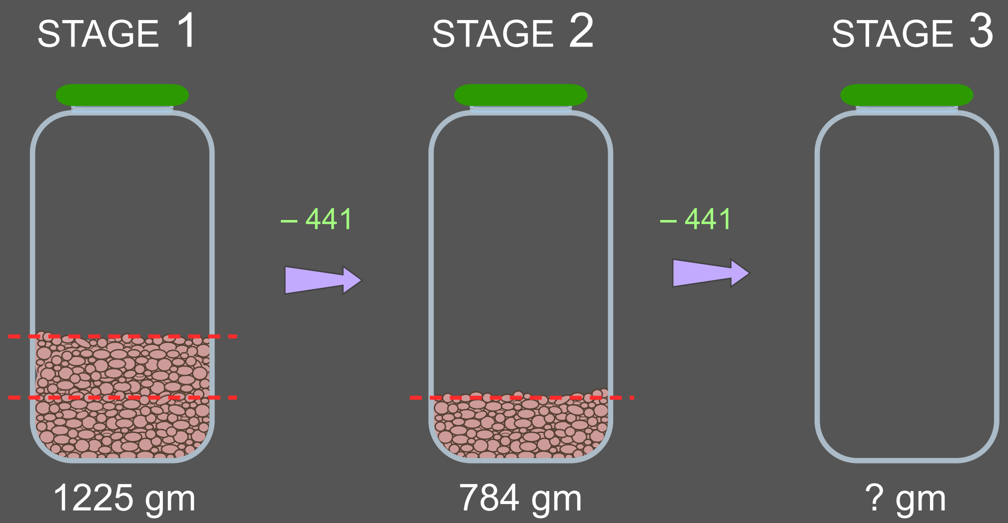


ans 18 completely nuts !

Where to begin? Well, why not draw a diagram?

this makes things much easier !

diagram shows the three stages of the jar and nuts



notice that we've used the same units for all the weights

going from stage 1 to stage 2, the jar loses half the nuts

. . . and that equals 441gm (1225 – 784 = 441)

going from stage 2 to stage 3, the jar again loses half the nuts

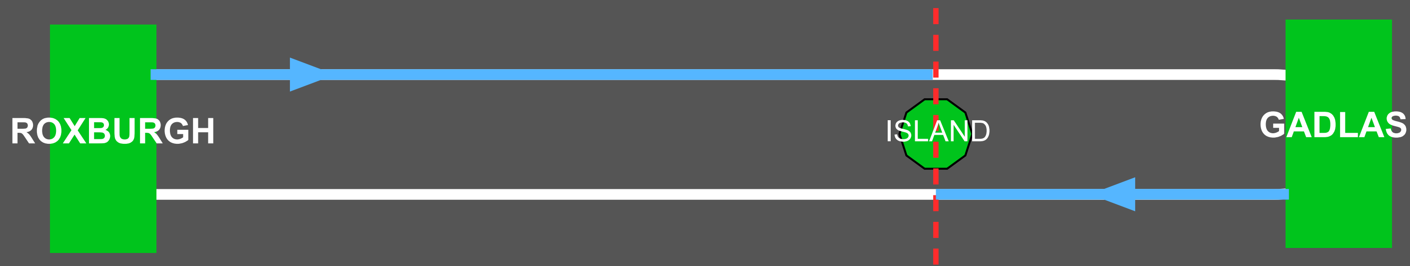
. . . so once again we lose 441gm

784gm – 441gm = 343gm = weight of empty jar

ans 19 Annabelle sails

This diagram shows the two parts of the race

Getting to the island took the same length of time each day, or in other words, the two stretches coloured blue took exactly the same time . . .



. . . but as Annabelle was travelling twice as fast on the first leg, she must have covered twice the distance. So Roxburgh to the island must be twice as far as Gadlas Creek to the island – which means that these two distances (adding up to 18 km, we know) must be 12 km and 6 km.

answer : the small island is 12 kilometres from Roxburgh

ans 20 dockyards & warships

To start with, let's simplify things by calling the four sailors A, B, C and D.

Next, you can anwer this question really easily if you spot the fact that whenever there are three men up on deck, there must be one man down below. In other words, every arrangement of three men on deck must correspond exactly to one particular man below deck. There are just four different ways of having a man below deck, so the answer to the problem is :

Starting with four men, you can make four different groups of three.

The four groups are

A B C / A B D / A C D / B C D

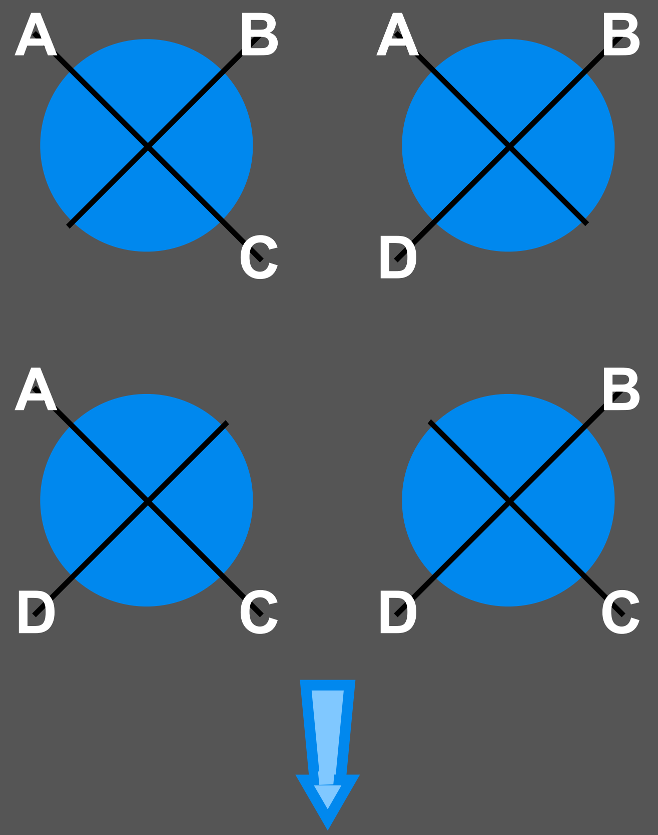
One way of getting these groups is to make a list with ABCD four times over and then to take out a different letter on each line (that's for the one man below deck). You can see this method illustrated here on the right :

A	B	C	
A	B		D
A		C	D
	B	C	D

PTO ⏩

Now here's another way of getting to the answer; this way will appeal to those of you who like to find a diagram approach to problems :

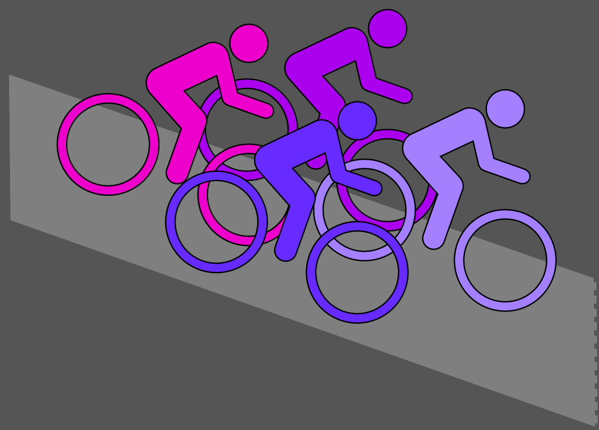
Imagine there's a four-seater table on deck where all four sailors can sit. One by one, you leave out one sailor (the one who must stay below deck) – and you end up with four different arrangements; that's to say, four different groups of three sailors :



A B C / A B D / A C D / B C D

ans 21 uphill & downhill

There are different ways of getting an answer here. You can list the downhill sections and record for each one how much descending is involved. Adding these figures together will give you the total amount of going down. Then you can list the ascending sections and write down for each one how much climbing is involved. Again, adding these together will give you a total, this time the total amount of climbing. Subtract one total from the other and you've got your answer!

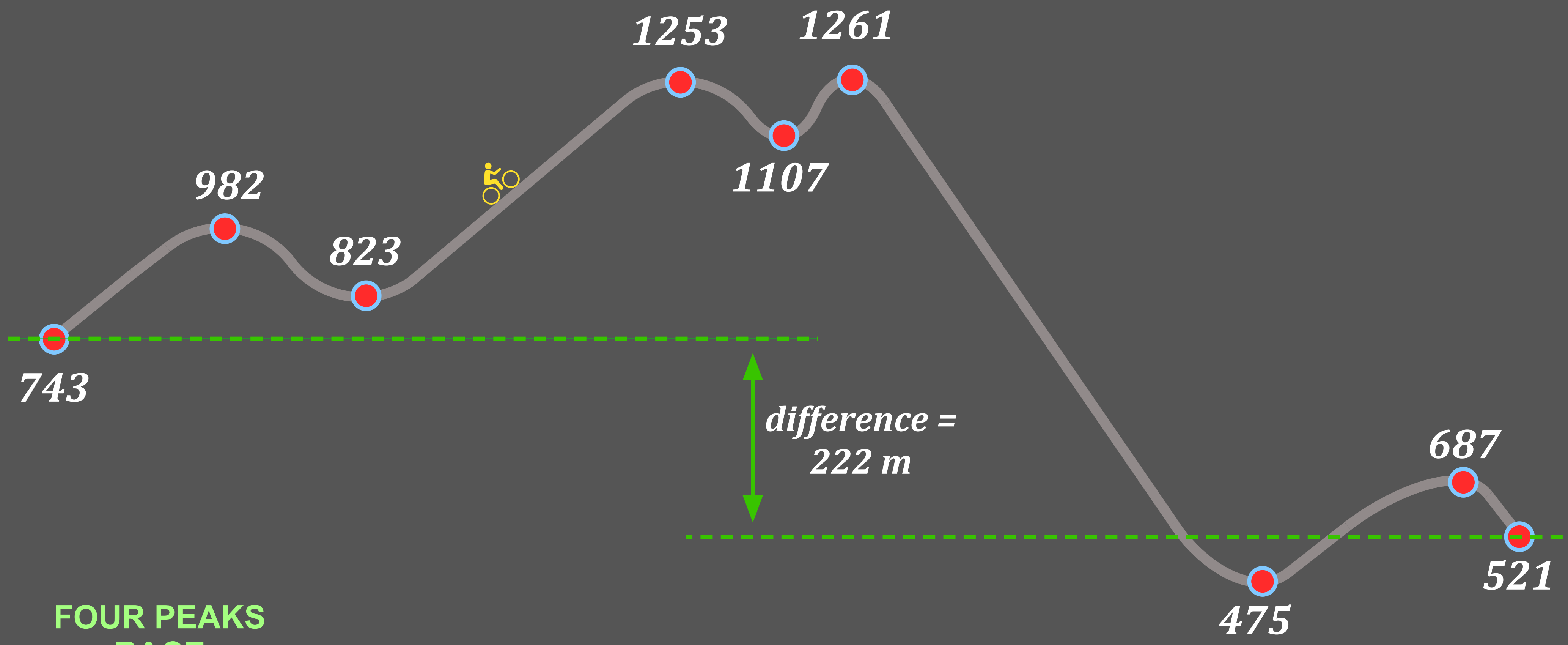


. . . Or, as the diagram (page following) makes clear, you can just take the height of the finishing point and the height of the starting point – and then simply subtract the one from the other . . .

Either way, your answer should be 222 m.

PTO ▶▶

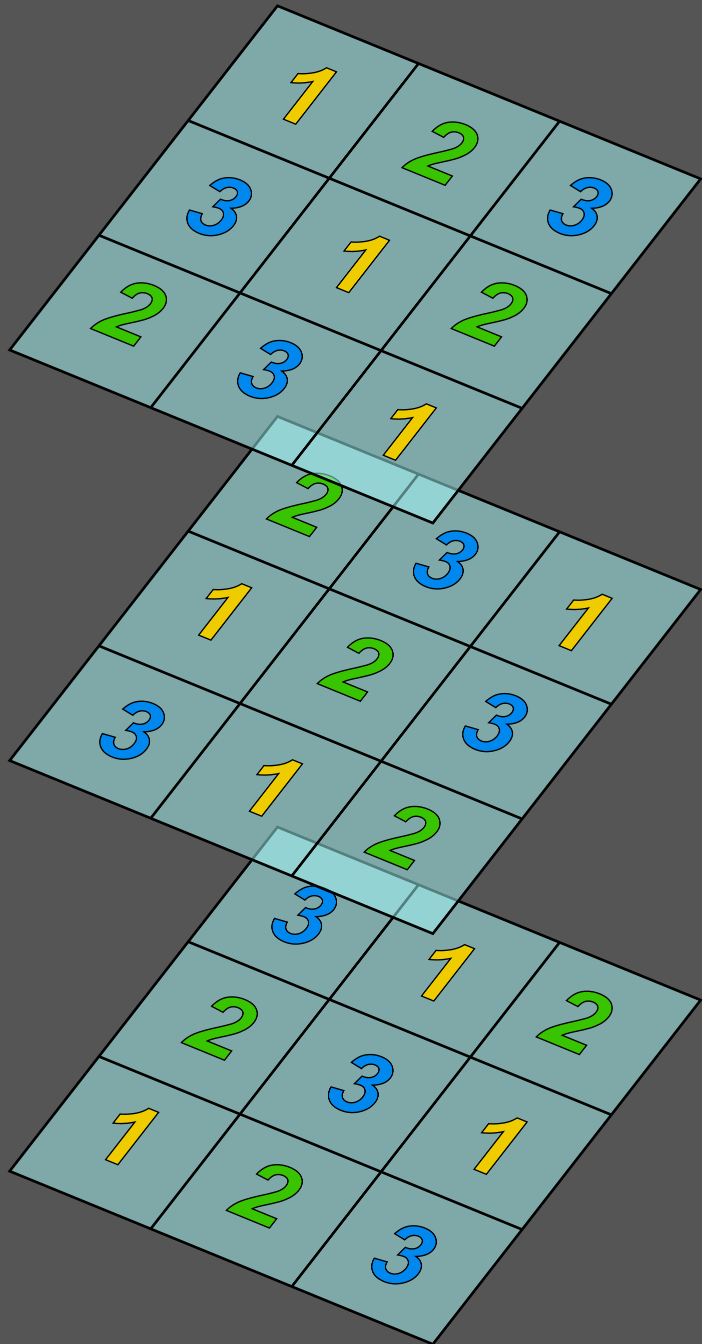
uphill & downhill



FOUR PEAKS RACE

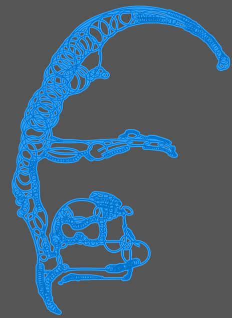
ans 22 latin cube

Here's one arrangement which works :



ans 23 just two gorillas

Perhaps the easiest way of solving this problem is just to pick a number which both 10 and 15 divide into. Let's choose 30, as that's the easiest. So, if we had 30 kg of food left, we could say :



Sultan will eat 30 kg in 10 days = 3 kg per day

Solomon will eat 30 kg in 15 days = 2 kg per day

so, Sultan + Solomon together will eat 3 + 2 = 5 kg per day

And with 30 kg to start with, 5 kg per day would last 6 days . . .

important note You don't have to choose 30 as your starting number – you could just as well choose 60 or 90 or any other number which both 10 and 15 divide into. Whichever of these numbers you choose, you'll get the same answer . . . try it !

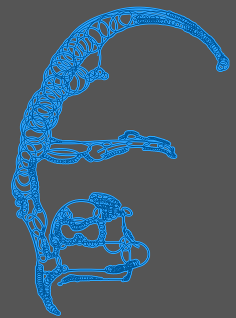
PTO

ans 23 just two gorillas

If you're comfortable with adding fractions, you might prefer to solve this problem more directly, like this :

Let's call the total amount of food we've got t kilograms. Then we can say :

$$\text{Solomon eats each day} = t/10$$



$$\text{Sultan eats each day} = t/15$$

$$\text{so together these two eat } t/10 + t/15$$

$$\text{and} \quad t/10 + t/15 = 3t/30 + 2t/30 = 5t/30 = t/6$$

– in plain English, this is to say that the two of them together will eat one-sixth of the remaining food each day . . . which means that this amount of food will last them exactly <u>6 days.</u>

ans 24 consecutive numbers . . .

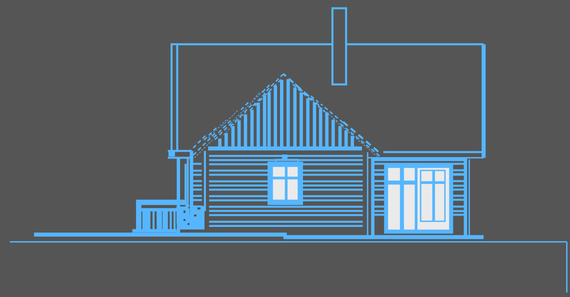
answers :

a. 15 can be made either from a pair of consecutive numbers (7 + 8) or from three consecutive numbers (4 + 5 + 6).

b. 30 can be made either from three consecutive numbers (9 + 10 + 11) or from four consecutive numbers (6 + 7 + 8 + 9).

c. No you certainly can’t ! If you add any two consecutive numbers, you’ll get an odd number – but if you add four consecutive numbers, you’ll always get an even number. No number can be both odd and even ! So there's no answer to this one . . .

the little house on the cliff . . .



20% of 100,000 is 20,000

So at first the value of the house fell by £20,000

100,000 – 20,000 = 80,000

In other words, after the bad news about unstable cliffs, the value of the house fell to £80,000.

The new geological survey led to an increase in value from £80,000 back to £100,000 – that's to say, an increase in value of £20,000

But 20,000 is 25% of 80,000.

So the answer is : When the value of the house returned to its original level, it went up by exactly 25%.

ans 26 sum of three primes

This question seems to be just about prime numbers but it's also about odds and evens . . . and the important thing to remember here is that when you're adding numbers, (1) *two odds or two evens make an even* and (2) *an odd and an even together make an odd* . . .

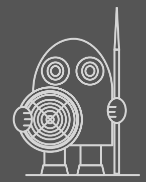
So, how does this affect our question? Well, we know that 100 is even, so to get this by adding three smaller numbers together, we must have either three evens or an even and two odds . . .

We know there certainly aren't three even prime numbers. So we must have two odd primes and an even prime. There is of course only one even prime number and that's 2. And 2 also happens to be the smallest prime number. So now we have our answer . . .

If three prime numbers add up to 100, the smallest of them must be 2.

extra question

With 2 as the smallest, what could the other two prime numbers be?



PTO

ANS 26 sum of three primes

With 2 as one of the three primes, the other two must obviously add up to 98. So we're looking for a pair of prime numbers which add up to 98.

The numbers we're looking for must be odd. But we also know that their last digits must add up to 8. All numbers ending in 5, apart from 5 itself, will be multiples of 5, so we needn't bother about them.(The only possible pair involving 5 is the pair 5 + 93 and that's no good because 93 is a multiple of 3.) So our only possible pairs must be chosen from :

either

number ending in 1 plus number ending in 7

or

number ending in 7 plus number ending in 1

Look at the list on the right. As you can see, most of them involve a multiple of 3, so they can't be a pair of prime numbers. What we're left with are the two pairs 37 + 61 and 67 + 31 and since all these four numbers are prime, we have our answer :

The other two primes must be either 37 & 61 or 67 & 31

98	
7 + 91	mult of 3
17 + 81	mult of 3
27 + 71	mult of 3
37 + 61	
47 + 51	mult of 3
57 + 41	mult of 3
67 + 31	
77 + 21	mult of 3
87 + 11	mult of 3

PTO

prime numbers

Here are the prime numbers up to 100. If you're aiming to become seriously good at maths, it's a good idea to learn off by heart at least the prime numbers up to 50. That's not really all that hard to do, especially if you start now . . .

2 3 5 7

11 13 17 19

23 29

31 37

41 43 47

53 59

61 67

71 73 79

83 89

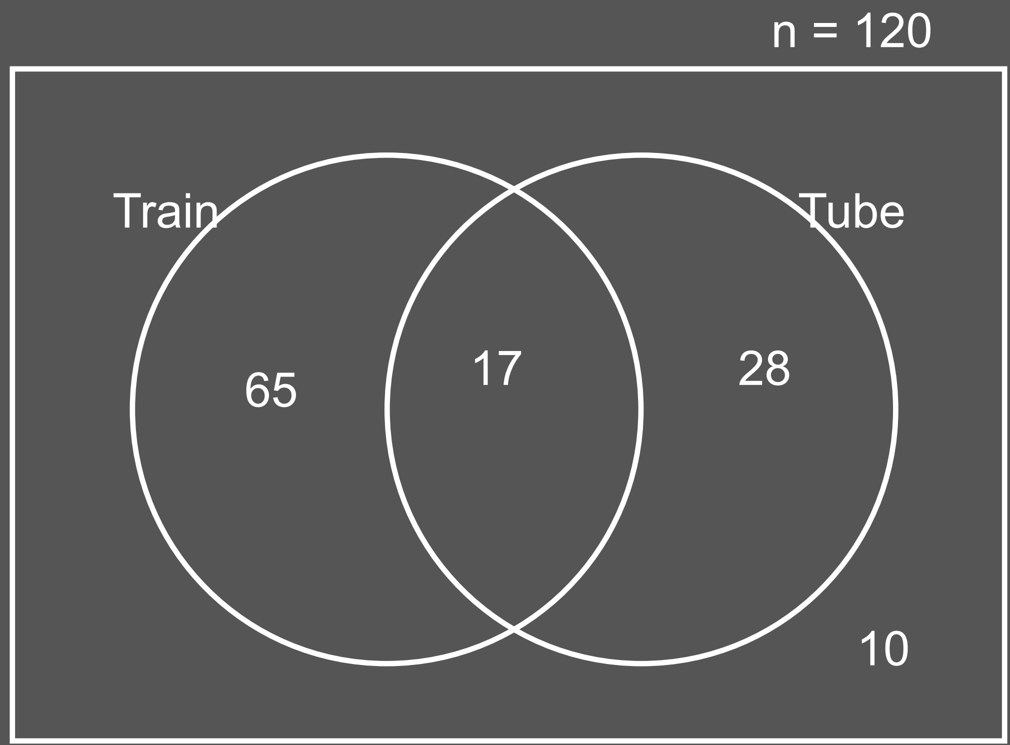
97

ans 27 transport for London

If you add together those who travel by train, those who travel by tube and those who don't travel by either, you get 137. But we know there are only 120 people in the survey.

We have 17 people too many! So, there must be 17 people we've counted twice – or in other words, there must be 17 people who travel by both train and tube. So, there must be 17 people in the intersection set (the set which, as you may know, we can write in symbols as Train ∩ Tube).

* And we know that only 65 people (82 – 17) travel by train alone and only 28 people (45 – 17) travel by tube alone.



$$\begin{array}{r} 82 \\ 45 \\ +\ 10 \\ \hline 137 \\ \hline \end{array}$$

$$\begin{array}{r} 137 \\ -\ 120 \\ \hline 17 \\ \hline \end{array}$$

answer : 17 people travel by both train and tube

 marking time . . .

You can't average the marks as they stand but if you turn them all to percentages, you'll have the same sort of thing throughout – and then you can work out an average in the usual way. (That's to say, add them all up and divide by 6.)

total of all six percentages = 420

so average percentage = 420 ÷ 6 = 70

And that's it : Anthony's average mark was a cool 70%. Not bad !

36 / 60	=	*60%*
48 / 50	=	*96%*
74%	=	*74%*
18 / 20	=	*90%*
30 / 75	=	*40%*
24 / 40	=	*60%*

If you're not sure about turning fractions into percentages, look on the next page . . .

PTO

ans 28 marking time . . .

36 / 60 .... cancel : divide top and bottom by 12, which will give you 3/5 ie 60%

48 / 50 .... just double the fraction and you get 96/100 ie 96%

74% .... it's already done for you!

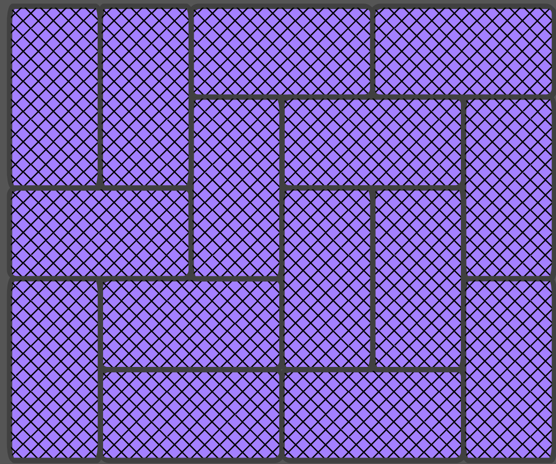
18 / 20 .... multiply top and bottom by 5 and you get 90/100 ie 90%

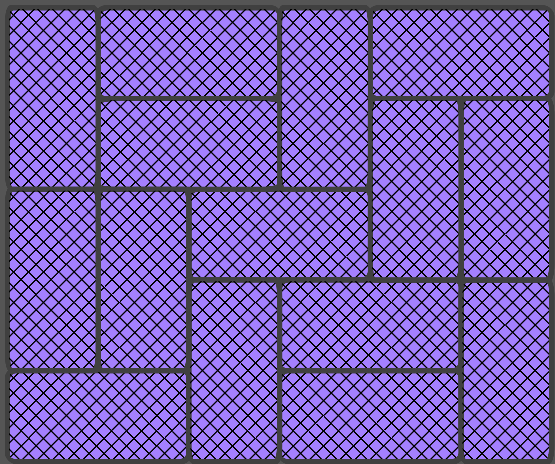
30 / 75 .... cancel : divide top and bottom by 15, which will give you 2/5 ie 40%

24 / 40 .... cancel : divide top and bottom by 8, which will give you 3/5 ie 60%

ans 29 fault lines

There are different ways of solving this problem – here's one way :



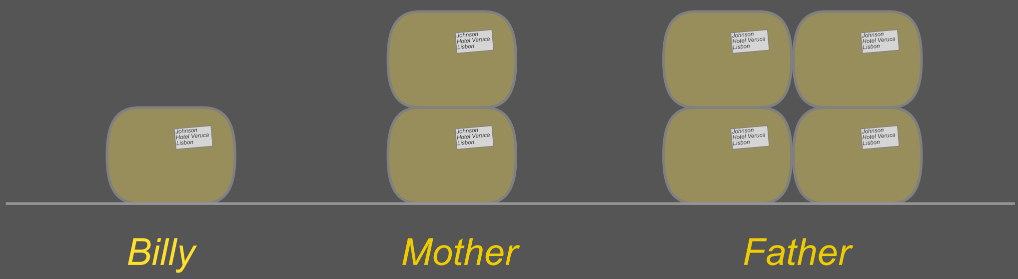


. . . and here's a different way

You might well have found other ways – but do double-check your answers : this is one of those annoying problems where you think you've solved it and then you look again – and see you've still got a fault line!

ans 30 an open and shut case

Forget the different shapes of these cases, just think of their weights :



From the information we're given, we know that Billy's luggage weighs half of Mother's luggage – and Mother's luggage weighs half of Father's luggage . . . So, using brown blobs to stand for units of weight, it's like our picture above, which shows : 1 unit for Billy, 2 units for Mother and 4 units for Father.

We're told that the difference between Father's luggage and Billy's luggage is exactly 150 kg – and we can see from the diagram that Father's luggage is 3 units more than Billy's. This means that the units are worth 50 kg each.

So we can say for definite that Billy's luggage weighs 50 kg, Mother's luggage weighs 100 kg and Father's luggage weighs 200 kg.

final answer : Mother's luggage weighs 100 kg

ans 31 what a racket!

To get to the bottom of this question, you really need to understand how to use 'byes' when you don't have the right number of players (eg 8, 16, 32, 64) to organise things simply. Below you can see how it works out when you do have the right number of players. As you can see, it's all pretty straightforward.

64 entrants

round 1 : 32 matches, 32 winners

round 2 : 16 matches, 16 winners

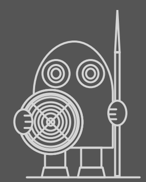
round 3 : 8 matches, 8 winners

round 4 : 4 matches, 4 winners

semi-finals : 2 matches, 2 winners

final : 1 match, 1 winner

On the following page you can see a diagram showing how you could use 'byes' to run a tournament when 11 players take part. Make sure you can see how this works.



PTO

ans 31 **what a racket!**

11 players enter

ROUND 1

5 players have a 'bye'

6 players play 3 matches (producing 3 winners)

what you're doing here in round 1 produces the same result for the next round (8 players) as if you'd started with 16

ROUND 2

8 players start this round : they play 4 matches (producing 4 winners)

SEMI-FINAL

4 players start this round : they play 2 matches and produce 2 winners

FINAL

2 players start this round : they play 1 match and produce 1 winner

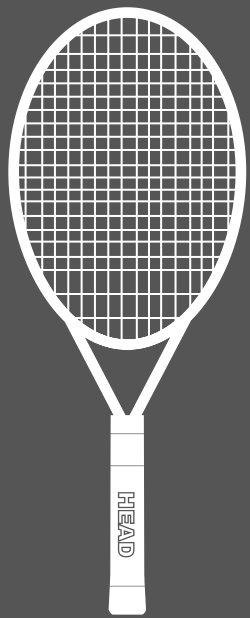
how to use 'byes' in a competition when you're starting with 11 players

PTO ▶▶

ans 31 what a racket!

By now you perhaps understand how the system of 'byes' works when you're organising a tournament and you don't happen to have eg 8, 16, 32, 64 players. But . . .

. . . you might have spotted something else too. Look back at the last two pages : how many matches *altogether* did it take to finally produce 1 winner? The answer is :



11 entrants – 10 matches in all

64 entrants – 63 matches in all

Yes, that's it! Whatever number of entrants you've got, just subtract 1 and that gives you the number of matches you'll need. Not really a surprise if you stop and think that each match played knocks out exactly 1 entrant.

So, coming back to the original question : Starting with 29 entrants, you'll need 28 matches in total to end up with 1 champion.

ans 32 late for work !

The most obvious way of solving this one is just to think of how many minutes past 7am it was when each of the clerks arrived. Add up all these minutes and divide by eight (because there were eight clerks) and you've got the mean (or average) time past 7am for the group :

$$\begin{array}{r} 20 \\ 35 \\ 26 \\ 22 \\ 36 \\ 27 \\ 37 \\ 29 \\ \hline 232 \\ \hline \end{array}$$

$$8\overline{)232}\ \text{ gives } 29$$

so mean = 29

answer : average arrival time was <u>07:29</u>

There's another way, which some people find easier – but you need to be happy working with negative numbers. This time you just write down how many minutes before or after 07:30 each clerk arrived and then average these numbers. The advantage of this method is that you're dealing with smaller numbers.

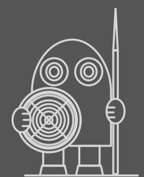
nb Use positive numbers for mins before 07:30 and negative numbers for mins after 07:30.

$$\begin{array}{r} 10 \\ -5 \\ 4 \\ 8 \\ -6 \\ 3 \\ -7 \\ 1 \\ \hline 8 \\ \hline \end{array}$$

$$8\overline{)8}\ \text{ gives } 1$$

so mean = 1 (that's 1 minute early)

answer : average arrival time was <u>07:29</u>



ans 33 tickets and pies

1 It should be easy enough to do this one in your head . . . but in fact many people reading the question and thinking of an answer quickly come up with this answer : 'The programme must have cost £1 and the ticket £10!' It's true that £1 and £10 add up to £11, but this answer won't do! Why not? Because the difference between the two amounts has to be £10 – so this time, the quick answer is a wrong answer! The right answer is of course : programme 50p, ticket £10.50 (These two amounts add up to £11 – and £10.50 is definitely £10 more than 50p, so we we can be sure this is the right answer).

What if you can't easily guess the answer? Is there a way of working it out? Well, you can think of this as an 'unfair sharing' problem – after all, you're just trying to share £11 between a programme and a ticket so that the ticket gets £10 more . . .

UNFAIR SHARING

remember to give out the 'unfair share' first . . .

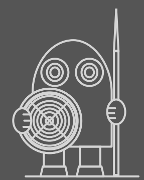
	programme	ticket
unf. sh.	0	10.0
f. sh	0.50	0.50
total :	0.50	10.50

PTO

ans 33 tickets and pies

2 Another 'unfair sharing' problem! This time you're sharing 47 mince-pies between the family and the School so that the School gets 13 more than the family. The answer is : family 17 pies, School 30 pies. As before, you start by giving the unf.sh (unfair share) of 13 to the School. This leaves 34 mince-pies, so you give a f.sh of 17 to the family and to the School. Then you add up to find the totals. Here's the working-out . . .

	family	school
unf. sh.	0	13
f. sh	17	17
total :	17	30



ans 34 a tricky question

Well, it might not be the smartest way of solving the problem – but one obvious approach is just to try some different numbers for the class size and see where it leads us . . .

FIRST ESTIMATE

Let's suppose the class size is 16. Then we can say there are 8 girls and 8 boys.

4 girls have their hands up, so 4 must have their hands down

Number of boys with hands down is 1.5 times the number of girls with hands down, so this number of boys must = 6

But we know that all the boys had their hands down, so number of boys in the class must = 6

We've already said that we're assuming 8 girls and 8 boys, so there's a contradiction! Class size = 16 just doesn't work!

What should our next estimate be? You'll probably agree that we need to try a larger number – and of course it has to be an even number. If you think for a moment, you might also see that all our estimates of class size will have to be multiples of 4 (otherwise we'll find ourselves with an odd number for 'girls with hands down' – and we son't be able to do 1.5 times that and get a whole number for the boys total. So our next few estimates should be 20, 24, 28 and so on . . .

PTO

ans 34 a tricky question

SECOND ESTIMATE

Let's suppose the class size is 20. Then we can say there are 10 girls and 10 boys.

4 girls have their hands up, so 6 must have their hands down

Number of boys with hands down is 1.5 times the number of girls with hands down, so this number of boys must = 1.5 X 6 = 9

But we know that all the boys had their hands down, so number of boys in the class must = 9

We've already said that we're assuming 10 girls and 10 boys, so again there's a contradiction! Class size = 20 doesn't work! But at least this estimate is better than the first!

THIRD ESTIMATE

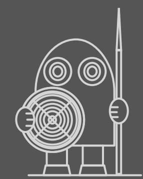
Let's suppose the class size is 24. Then we can say there are 12 girls and 12 boys.

4 girls have their hands up, so 8 must have their hands down

Number of boys with hands down is 1.5 times the number of girls with hands down, so this number of boys must = 1.5 x 8 = 12 and so there must be 12 boys altogether in the class

So with this estimate, all the figures work out nicely! And so at last we have our answer :

Altogether, there were 24 pupils in Form 3N.

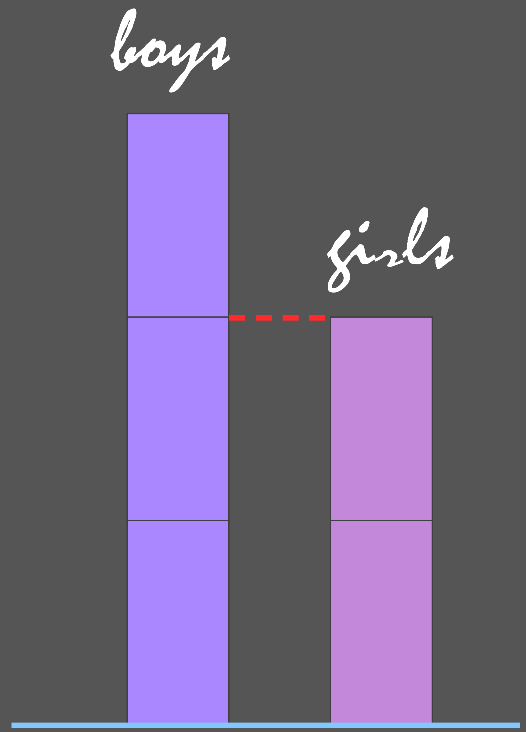


PTO ▶▶

Of course, some people would prefer a diagram as a good way to see what's happening here . . .

We know that one and a half times as many boys as girls had their hands down. On the right is a simple way of showing this fact :

* We could call this the 'hands down' diagram.



PTO

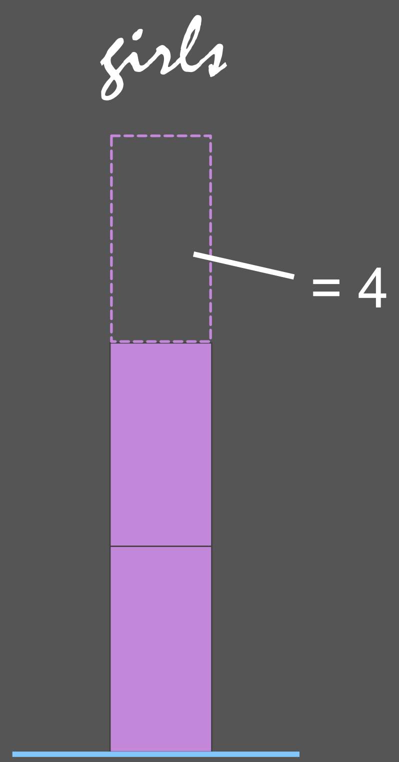
ans 34 a tricky question

In the first diagram we used 3 rectangles to stand for all the boys and just 2 rectangles to stand for all the girls. The rectangle missing on the girls' side stood for those girls whose hands were up.

In the diagram on the right we've shown the 'hands up' girls by an empty rectangle with a dotted line around it. There were 4 girls with their hands up, so of course this rectangle must stand for 4 girls.

This means there must have been 12 girls in all (three rectangles!). There were exactly the same number of boys as girls, so once again there's our answer :

Altogether, there were 24 pupils in Form 3N.



PTO

a tricky question

Of course, we don't actually have to have diagrams; we can reason our way to an answer without them. Some of you might well prefer this way of getting from the info we're given to the answer we're after. Here's a purely verbal way of arguing it out :

$1\frac{1}{2}$ times as many boys as girls had their hands down

so we could say $\frac{3}{3}$ of the boys had their hands down and $\frac{2}{3}$ of the girls had their hands down

or in other words, $\frac{1}{3}$ of the girls had their hands up

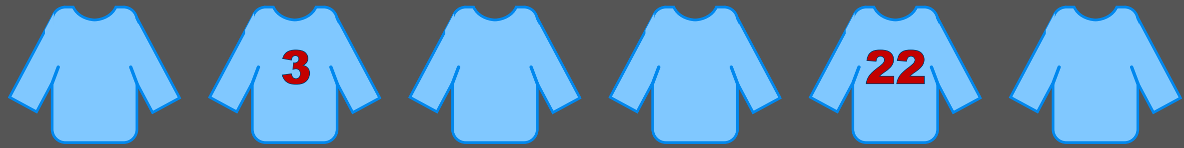
but we know that girls with hands up equals 4 in number; that's $\frac{1}{3}$ of the total number of girls, so this total must equal 12

. . . all of which means total class size = 24

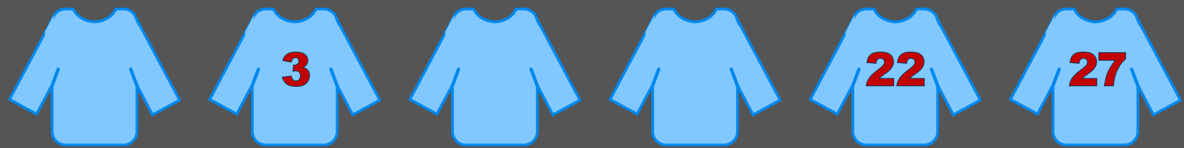
ans 35 6-a-side football

Let's go through the facts we're given and see what we can work out :

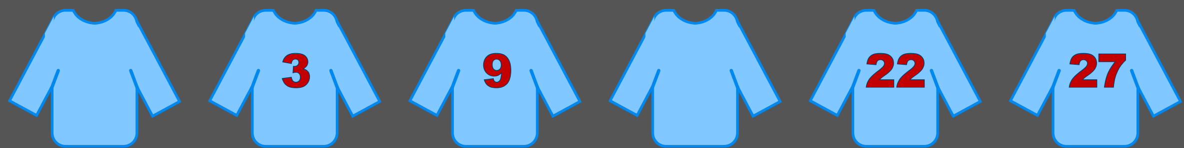
fact 1 tells us that the second number is 3 and the fifth number is 22. So that's a good start :



the fifth number is 5 less than the sixth number (fact 4) : so the sixth number must be 5 more than the fifth, or in other words, it's 27 :



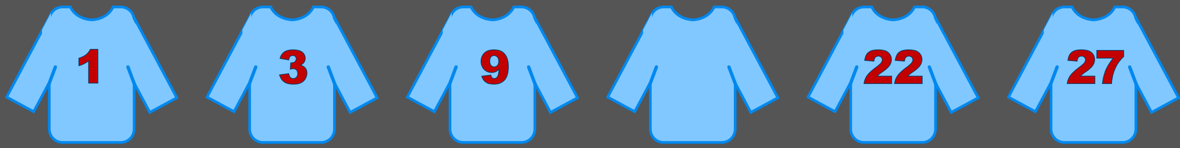
if you square the second number, you'll get the third (fact 3) : that's easy, we all know that $3^2 = 9$



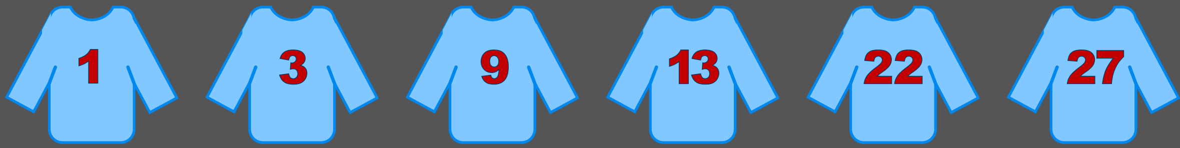
PTO ▶▶

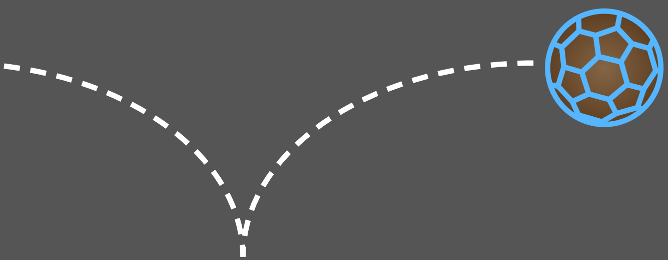
ans 35 6-a-side football

fact 5 tells us that the first three numbers add up to 13 : 3 and the 9 add up to 12, so the missing (first) number must be 1 :



the first four numbers add up to 1 less than the sixth (fact 2). So the first four numbers must total 26, meaning the fourth has to be 13 :





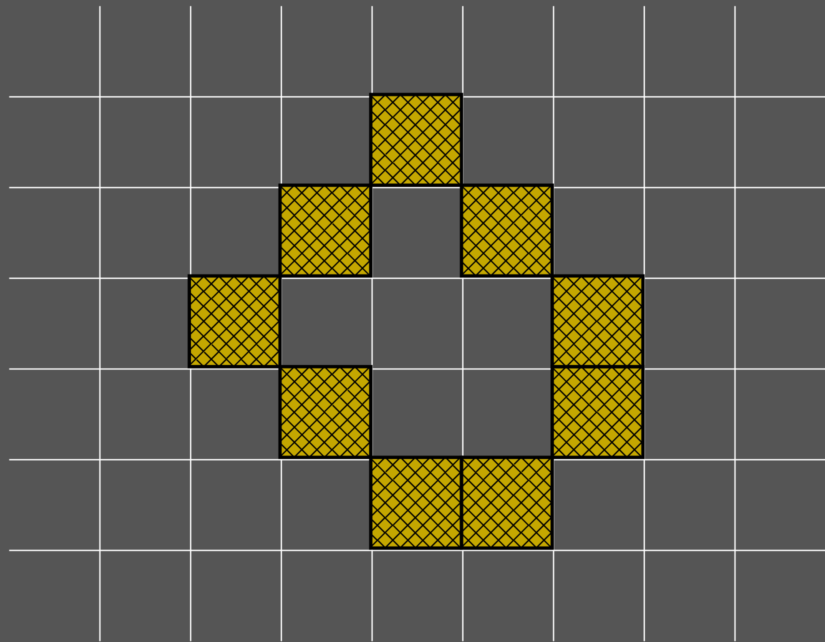
So there we are! The six numbers on the 6-a-side team shirts were 1, 3, 9, 13, 22 and 27.

ANS 36 sheep may safely graze

ANSWER : 6 square metres (or 6 squares) seems to be the maximum area you can enclose. Here's one way of doing it :

. . . maybe this is the only way of doing it !

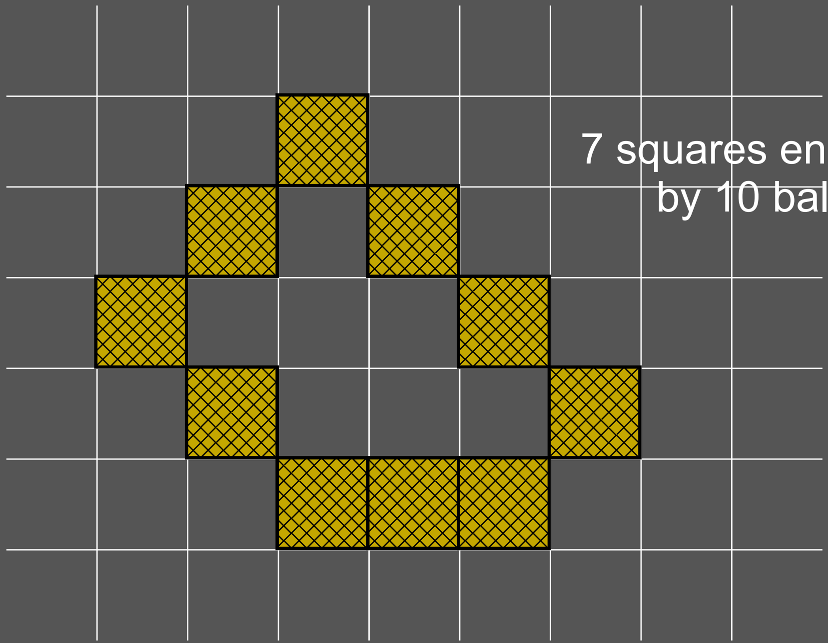
. . . and if you're looking for more sheep pen challenges, try to find a way of enclosing 7 squares using 10 bales – and then perhaps a way of enclosing 8 squares using 10 bales . . .
(answers on the next page)

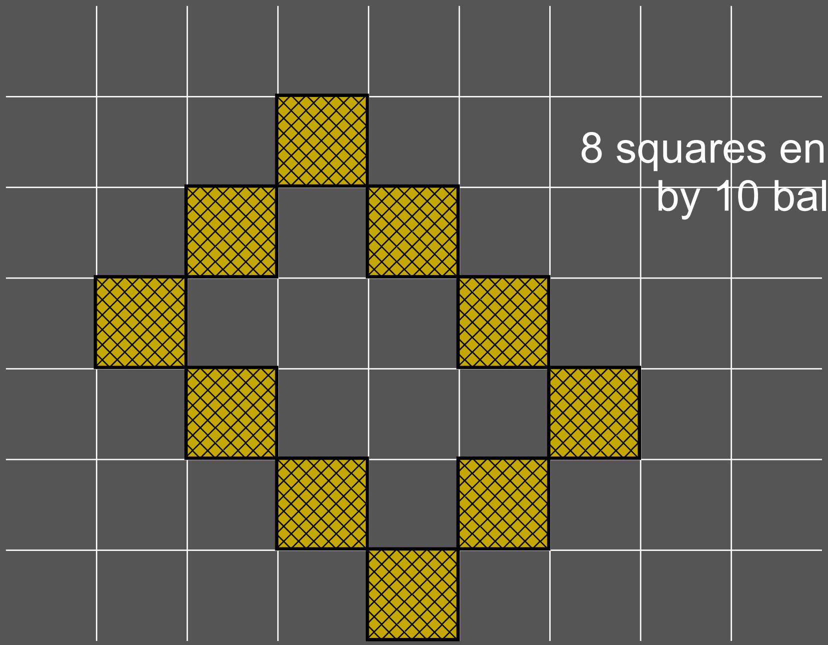


PTO

ans 36 **sheep may safely graze**

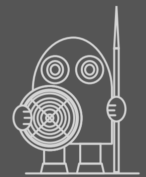
Here are possible answers for the two challenges we gave you on the previous page :





PTO ⏩

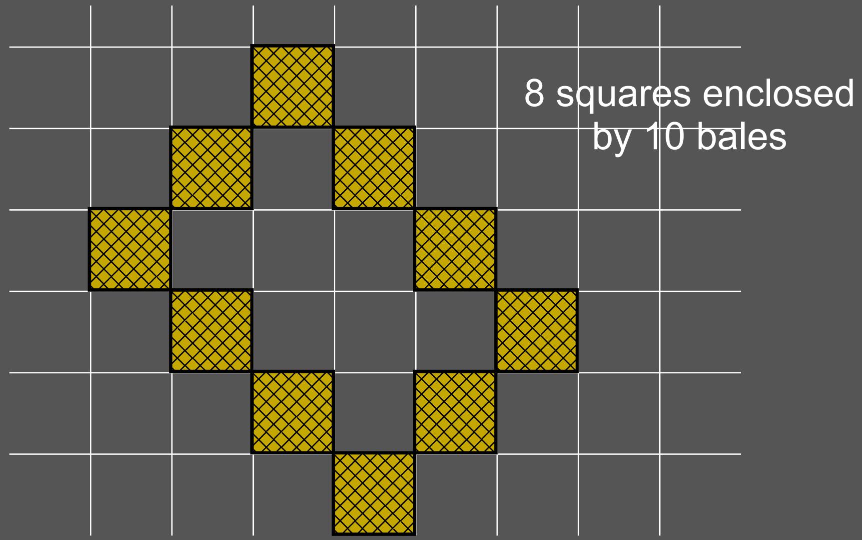
for two further challenges



ans 36 sheep may safely graze

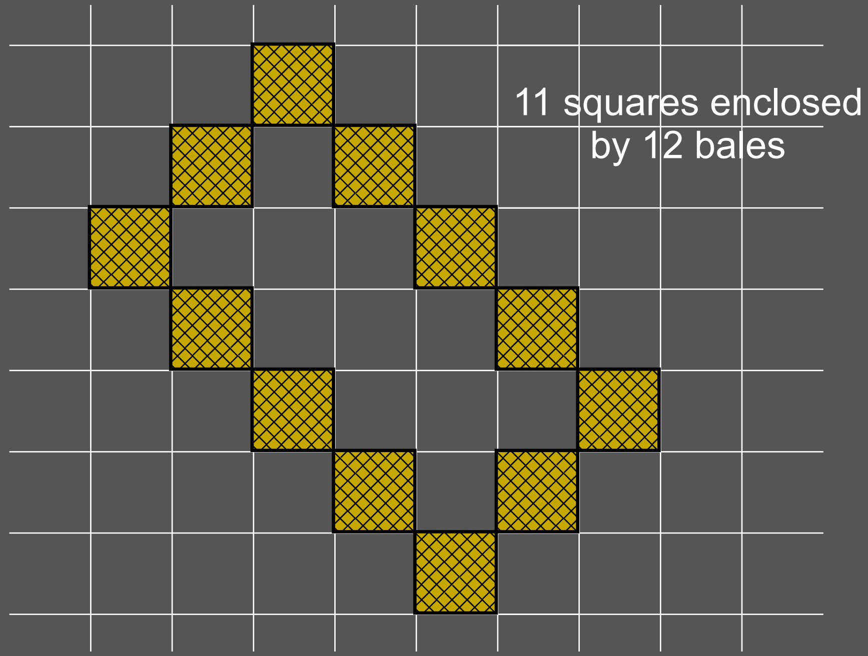
Here we have 10 bales arranged so as to enclose 8 squares. Could you arrange the 10 bales differently so as to enclose 9 squares?

If you think this can't be done, can you suggest a reason why it can't be done?



Now, here's a similar arrangement, this time with 12 bales arranged so as to enclose 11 squares. Do you think there would be any way of arranging the 12 bales differently so as to enclose 12 squares?

Say why you've answered yes or no to the question above.



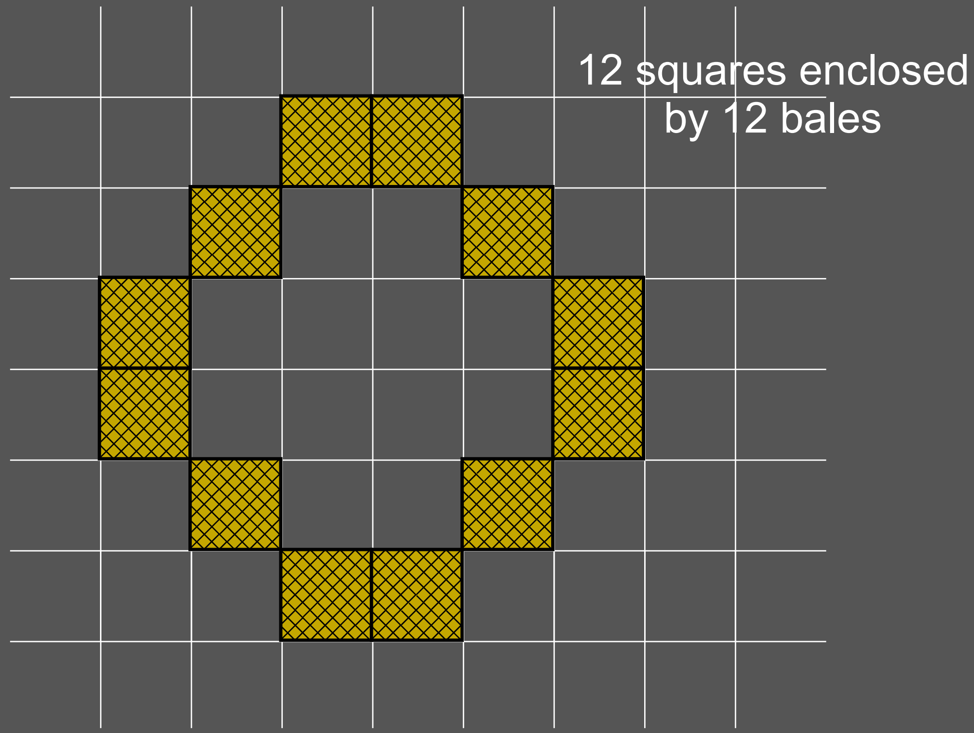
PTO

ans 36 sheep may safely graze

Well, this time it can be done, and here's how :

The difference is that the shape we've enclosed here is nearer to a circle – and any mathematician will tell you that for example if you have a given length of rope, the greatest area you can enclose is by arranging the rope in a circle.

So – just arranging all the bales corner to corner doesn't guarantee that you'll be enclosing the greatest area! You also need to think about getting your bales as close to a circle as you can . . .



ans 37 Pablo's progress

To sort this out, we need to be clear what the percentages are percentages of. And that's not too hard once you think about it. For example, the 85% Pablo was given for mental maths just means that he got 85% of the available 20 marks. So we need to calculate 85% of 20 – and that gives us his actual score in the Mental Maths section. This is how it works for the whole maths paper :

85% of 20 = 17 . . . 17/20

90% of 30 = 27 . . . 27/30

86% of 50 = 43 . . . 43/50

Adding up these three marks gives us a total of 87 out of 100. So that's it :

Pablo's overall maths percentage = 87%

ans 38 happy birthday James !

There are different ways of going about this problem; one easy way is to try some different possible answers until we find what we're after but . . . before we start trying lots of numbers, let's stop and think : If next year, Sophie's age is three times James' age, then Sophie's age must be a multiple of 3. So let's try some multiples of 3 for Sophie and next to them we'll put the corresponding ages for James and after that, their ages last year.

	Sophie	James
next year →	3	1
last year →	1	–

	Sophie	James
next year →	6	2
last year →	4	–

	Sophie	James
next year →	9	3
last year →	7	1

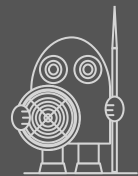
	Sophie	James
next year →	12	4
last year →	10	2

	Sophie	James
next year →	15	5
last year →	13	3

	Sophie	James
next year →	18	6
last year →	16	4

This took a time : we had to wait until our sixth attempt to find numbers which worked out. So finally, if Sophie is 18 next year and James is 6 next year, it all works. Success!

So, that's our answer : next year James will be 6.



ans 39 new year neighbours

How to get started with this problem? You could just keep thinking of numbers which might work and then each time look at the next numbers above and below . . . but that could turn out to be a very slow process. We need a definite way to go about things.

One idea is to start off with numbers in one of the sets involved and look at their neighbours. The square numbers are a good starting point – we know that there are only seven of them below 50 :

1, 4, 9, 16, 25, 36, 49.

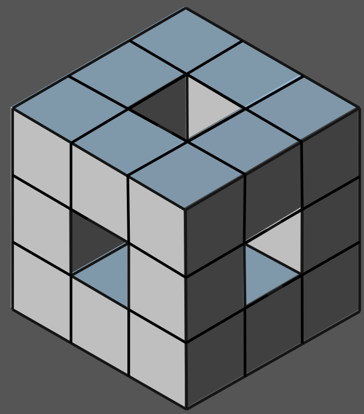
It doesn't take long to spot where the 'special' numbers lie. On the right you can see the list we've made.

answer : 3, 8, 10, 24, 48, 50

PRIME		SQUARE		
		1		
2	3	4		
7	8	9		
		9	10	11
		16		
23	24	25		
		36		
47	48	49		
		49	50	51

ans 40 the heartless cube

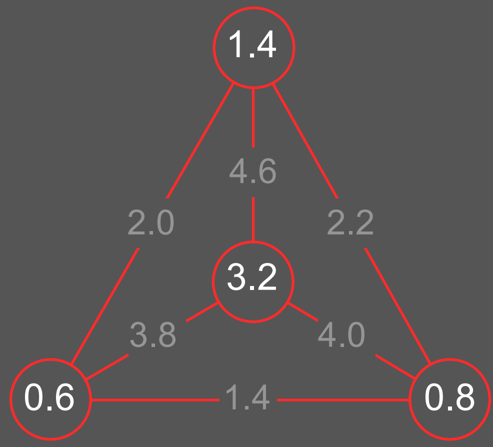
a. There are 20 1cm cubes in the shape. You can get this answer either by looking at the diagram and counting carefully – or you can just say there are 27 small cubes altogether in a complete 3 x 3 x 3 cube and this one has 7 cubes missing : that's 1 cube missing from each face (6 cubes in all) and 1 cube missing from the centre, leaving us with 20 cubes.



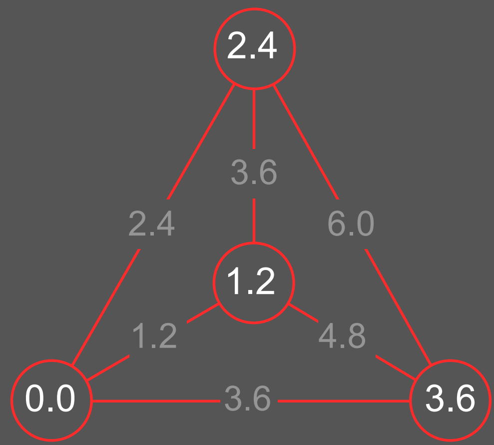
b. The total surface area of the shape is 72 cm^2 (six faces, each with 8 squares showing on the outer surface and six 'holes', each surrounded by four 1cm squares).

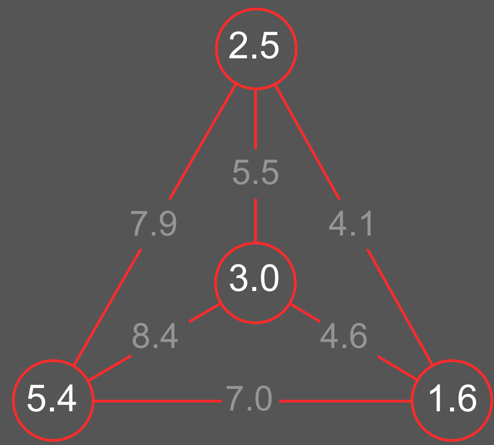
ans 41 more number triangles

Here are the answers to these three difficult number triangles : the hard way to solve them is just to keep trying numbers until you have some which work – but, as you may have discovered in the easier number triangle questions, a simpler way is to choose one triangle within the shape and then solve that triangle.



* On the next page there are two examples, an easy number triangle and then a harder one, with which we show you how to get started on filling in the blanks . . .

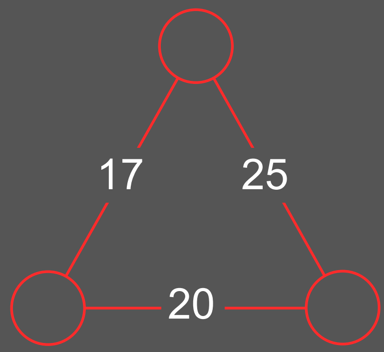




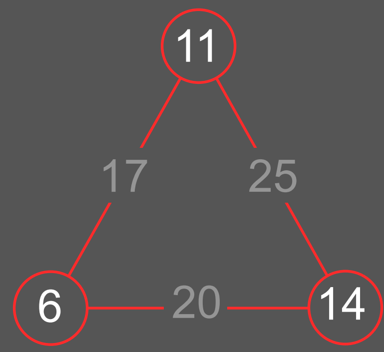
note : some people find it easier here to turn decimals like these into whole numbers – and then to turn them back to decimals at the end.

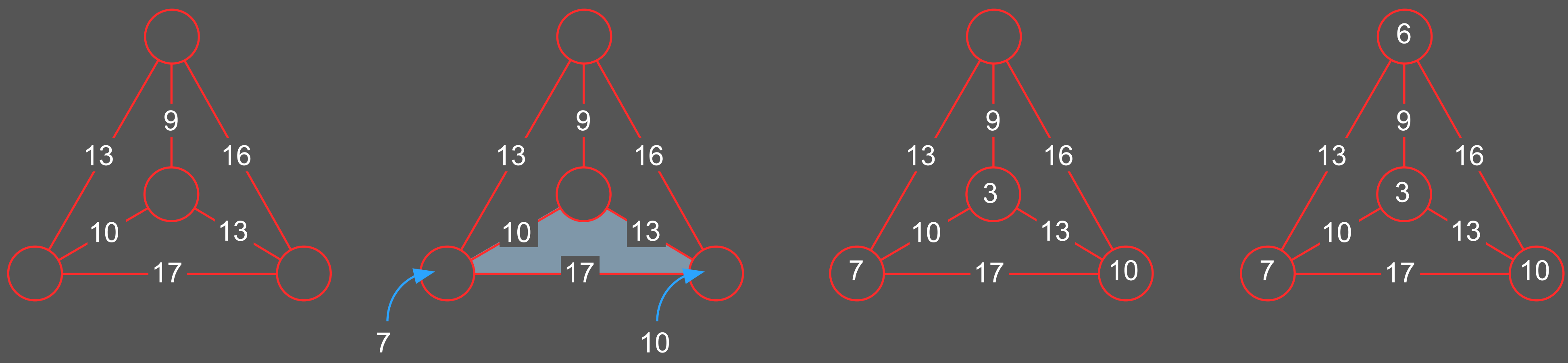
PTO

ans 41 more number triangles



Look at the bottom row : we need two numbers which add up to 20. We can see that 25 is 8 more than 17, so we're looking for two numbers where one is 8 more than the other – can you see why? It's easy then to see we need 6 and 14 :





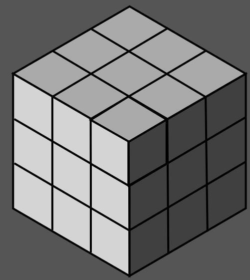
Now let's use the same method for the this difficult one . . . Let's choose the bottom triangle, the one we've shaded blue. For the bottom circles, we need two numbers which add up to 17 – and which are just 3 apart from each other. 7 and 10 fit the bill (get them the right way round!), so we put them in . . . and immediately we can put 3 in the top circle to complete the bottom triangle. Now we have enough information showing to put 6 on top and complete the whole thing.

ans 42 the missing cube

- Each face of the cube has an area of 3 x 3 = 9cm²

  And the large cube has 6 of these identical faces

  So, surface area of the large cube = 6 x 9 = 54cm²



- One way of going about problem 2 is as follows :

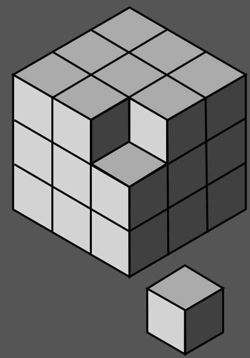
  Large cube with one missing corner cube =

  3 faces each with area 9cm2 = 27cm²

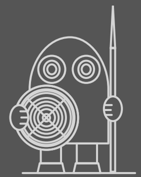
  3 faces each with area 8cm2 = 24cm²

  3 small faces each 1cm2 = 3cm²

  54cm²



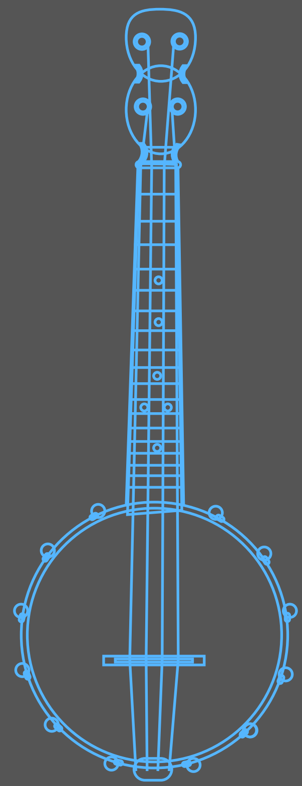
- Perhaps a neater way of solving problem 2 is this : Notice that when you remove one corner cube, you take away three 1cm² outer faces – but at the same time you expose three 1cm² faces which were hidden before. So final result = same as before = 54cm²



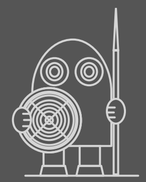
ans 43 the frog and banjo

There are different ways of going about this problem but here's one way you might like. Begin by making a table like this :

	Frog	*Newt 1*	*Newt 2*	*Toad*
banjo				
lead guitar				
rhythm guitar				
double-bass				



. . . and then work through the information you're given, putting red circles to show where something isn't possible and green circles to show where something must definitely be so . . .



PTO

ans 43 the frog and banjo

First of all, Newt 1 did not play double-bass. So, put in a red circle to show this.

Next, we know that Toad didn't play either lead guitar or rhythm guitar, so put in red circles to show this.

And we do know that Frog played the banjo, so put in a green circle to show this.

Next, we're told that Newt 1 didn't play rhythm guitar. So put in a red circle to show this.

Finally, we know that Newt 2 didn't play lead guitar, so put in a red circle to show this.

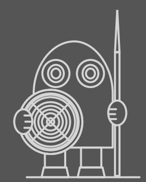
	Frog	*Newt 1*	*Newt 2*	*Toad*
banjo	🟢			
lead guitar			🔴	🔴
rhythm guitar		🔴		🔴
double-bass		🔴		

That takes care of recording the information we're given. The next step is to do some thinking :

1 If Frog played the banjo, he couldn't have played any of the other instruments, could he? So add three red circles to show this.

2 And of course, if the banjo was played by Frog, then it obviously wasn't played by any of the other three band members – so put in three red circles to show this

	Frog	*Newt 1*	*Newt 2*	*Toad*
banjo	🟢	🔴	🔴	🔴
lead guitar	🔴		🔴	🔴
rhythm guitar	🔴	🔴		🔴
double-bass	🔴	🔴		



PTO ▶▶

ans 43 the frog and banjo

And we're almost there! We know what Frog played but if we look down the column for Newt 1, we can see three red circles and one blank space (level with lead guitar). Newt 1 must have played lead guitar; so put in a green circle to show this.

If we look further along, we can see the same thing applies to Toad : there are three red circles in his column and one blank space. The blank space is level with double-bass, so Toad must have played double-bass. Put in a green circle to show this.

	Frog	Newt 1	Newt 2	Toad
banjo	🟢	🔴	🔴	🔴
lead guitar	🔴	🟢	🔴	🔴
rhythm guitar	🔴	🔴		🔴
double-bass	🔴	🔴		🟢

And finally . . . there's only one player (that's Newt 2) without an instrument and there's only one instrument (rhythm guitar) without a player. So obviously Newt 2 must have played rhythm guitar . . .

. . . and our table is complete !

	Frog	Newt 1	Newt 2	Toad
banjo	🟢	🔴	🔴	🔴
lead guitar	🔴	🟢	🔴	🔴
rhythm guitar	🔴	🔴	🟢	🔴
double-bass	🔴	🔴	🔴	🟢

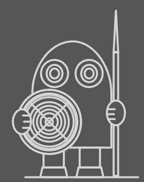
FINAL ANSWER :

Frog played banjo

Newt 1 played lead guitar

Newt 2 played rhythm guitar

Toad played double-bass



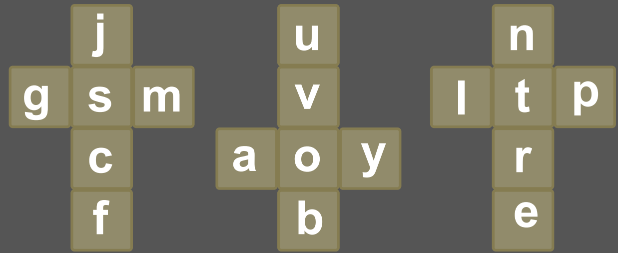
ans 44 cube calendar - months

we found an answer – but how did we get it?

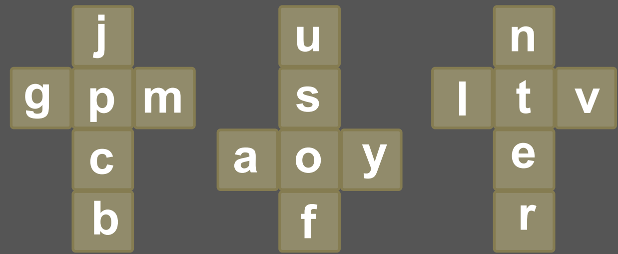
a b c d e
f g h i j
k l m n o
p q r s t
u v w x y z

19 letters needed – so perhaps not too hard to fit onto 3 cubes (18 faces) – especially as some can double up ('d'and 'p' or 'u' and 'n')

we started off with a trial-and-error approach :



<<— this one can't do 'april'



<<— this one can't do 'may'

. . . but soon decided a more systematic approach was needed . . .

PTO

ans 44 cube calendar - months

First of all we made a vertical list of all 12 months of the year, using just 3-letter abreviations : jan, feb, mar . . . and so on.

j	a	n
f	e	b
m	a	r
a	p	r
m	a	y
j	u	n
j	u	l
a	u	g
s	e	p
o	c	t
n	o	v
d	e	c

Then we looked at our list and started swapping letters around to try to get every letter a in one column, every letter j in another column, every letter p (= d) in another column . . . and so on. Here's the vertical list we ended up with :

a	j	n
f	e	b
a	r	m
a	r	p
a	y	m
u	j	n
u	j	l
a	g	u
s	e	p
c	o	t
v	o	n
c	e	d

Now we knew that letter v had to go onto the first cube, letter j had to go onto the second cube and letter l had to go onto the third cube. Next, we knew that a, e and p had to go onto the first , second and third cubes respectively. And in this way we continued . . .

v	j	l
a	e	p
f	r	b
u	y	n
s	g	m
c	o	t

So, here's a plan of the three cubes, showing where we put the letters on each. (Notice that u and n had to be on separate cubes, just because of jun)

	v	
a	f	u
	s	
	c	

	j	
	e	
r	y	g
	o	

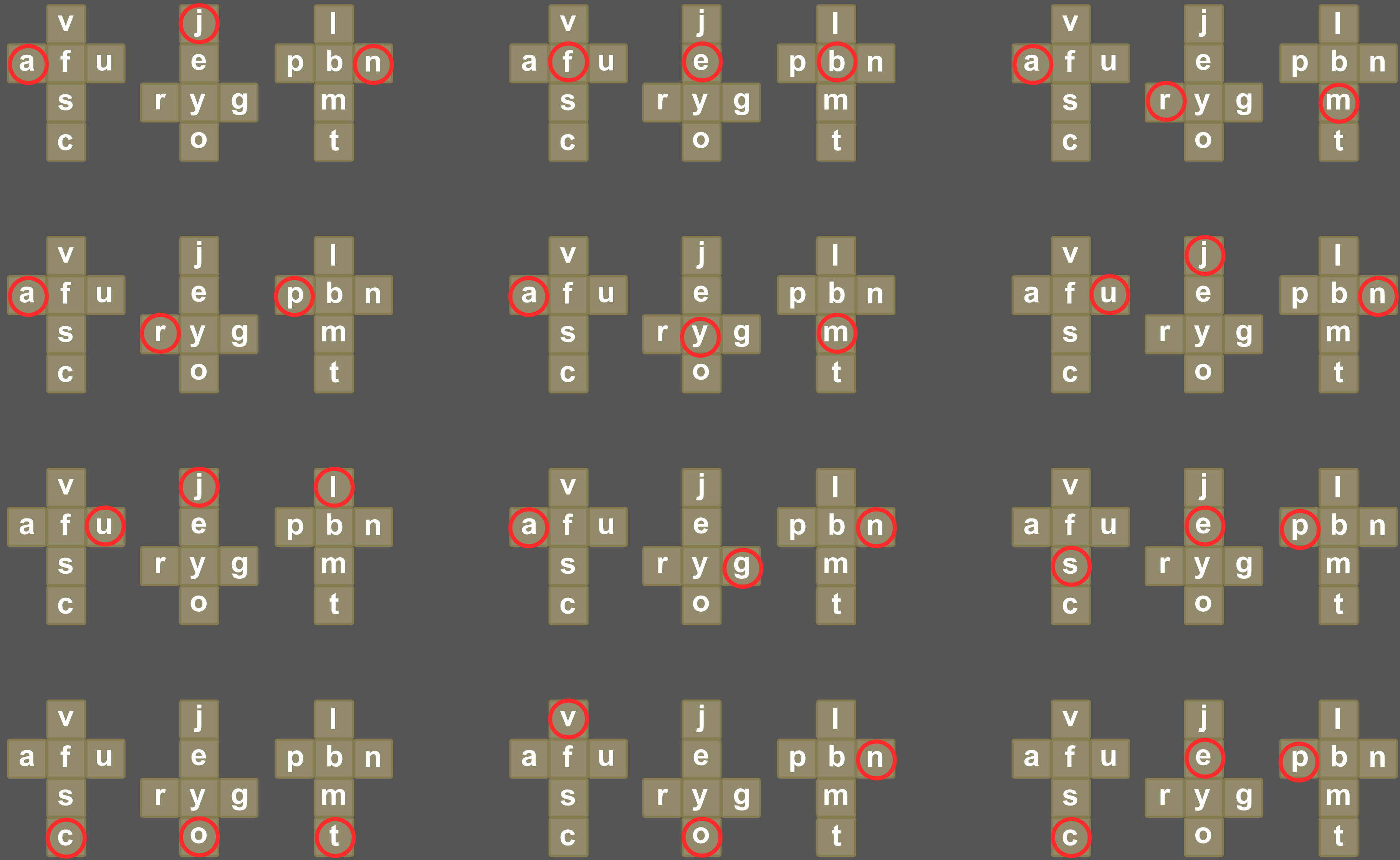
	l	
p	b	n
	m	
	t	

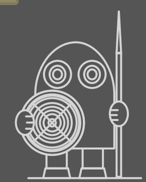
This is just one way of arranging the letters on the three cubes : perhaps you found a different arrangement? And perhaps you used a different approach? The following page shows how our arrangement can be used to give you all 12 months of the year.

As mentioned in the question, you have to use a bit of cunning with some months : to get 'aug' you have to use the 'n' on one brick and turn it upside down – and to get 'dec' you have to use a 'p' turned upside-down.

PTO

ans 44 cube calendar - months



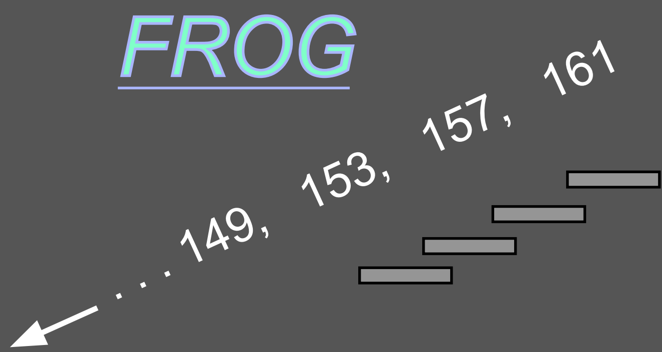


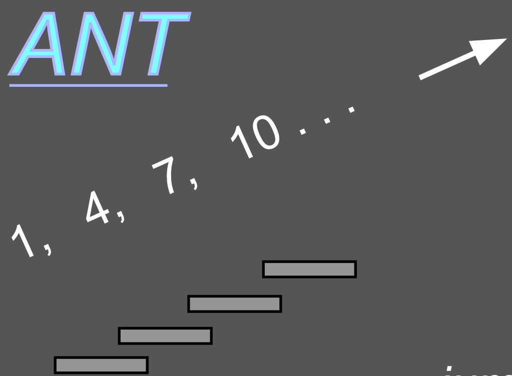
PTO

ans 44 cube calendar - months

. . . and finally, if you want to make a cube calendar of your own, you'll need five wooden cubes and some self-adhesive letters and numerals (Mr Pascal used Letraset Helvetica self-adhesive letters and numerals). Here's a layout which works :

ans 45 stairway to heaven

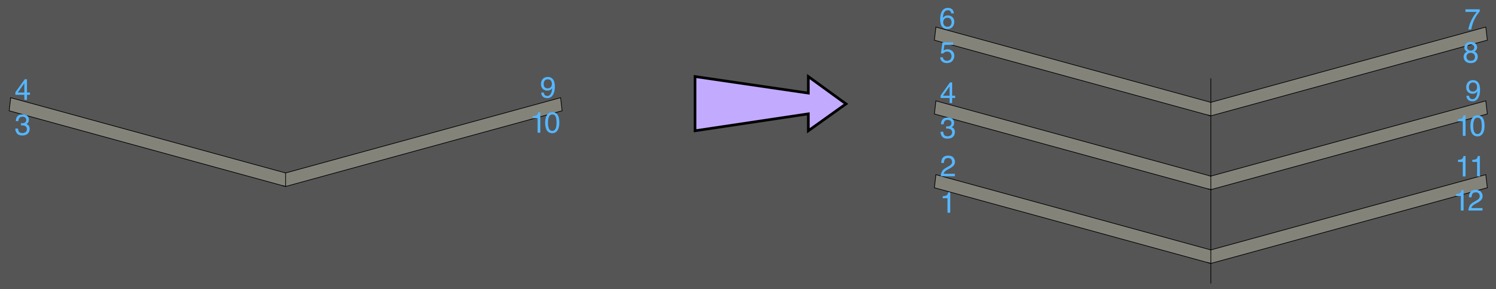




The picture gives you an idea of how things are for the first few steps – the numbers show you clearly which step the ant (down at the bottom) and the frog (up at the top) are on at any point. Remember, the two of them start off at exactly the same time. Then when the clock chimes, they each jump. The ant jumps up by 3 steps each time; the frog jumps down by 4 steps. This means that each time they jump, the two of them get 7 steps nearer together. This tells you that that if the number of steps between them at any time is an exact multiple of 7, then the two creatures will surely meet. Luckily 161 – 1 = 160, which is not a multiple of 7. So . . . the ant seems likely to survive – at least for the time being . . .

ans 46 after the flood

One way of solving these two problems is just bit-by-bit to reconstruct the original booklets. It's easy to see with the 4 and 9 double-page, with 3 and 10 on the other side, that the sheet which used to lie underneath this one would have had 2 and 11 on the one side and 1 and 12 on the other side. And it's not hard to picture the remaining top sheet – with 6 and 7 on the one side, coupled with 5 and 8 on the other side :



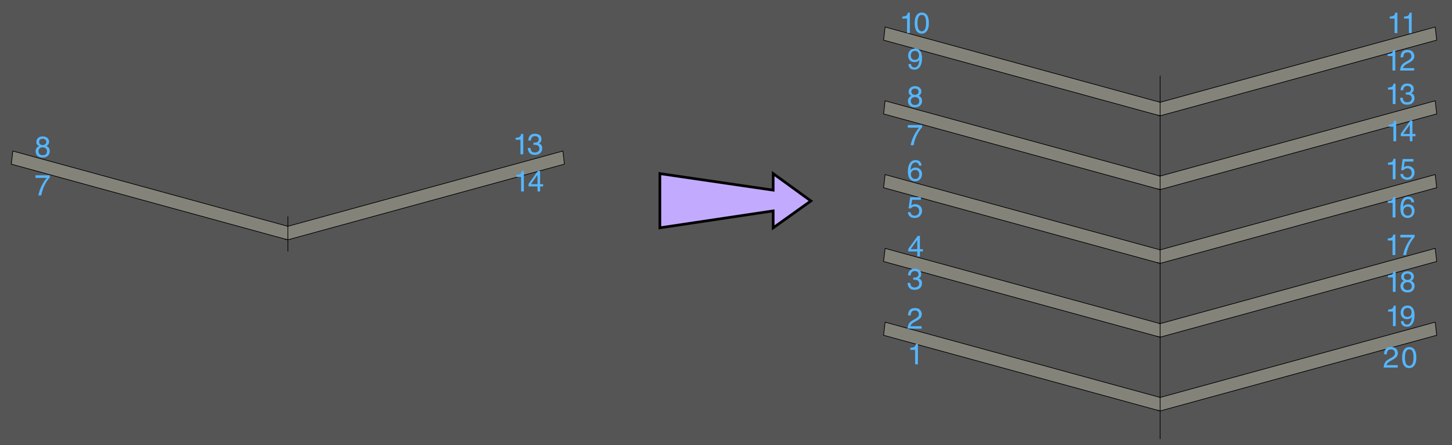
So, as you can see, there were 12 pages altogether in the original booklet – and the booklet was made from three single A4 sheets.

** Did you notice that the page-numbers facing you on any one sheet always add up to 13? Perhaps something to remember . . .*

PTO

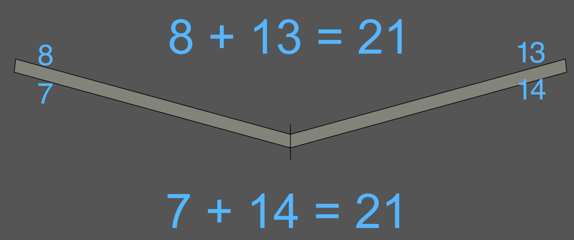
ans 46 after the flood

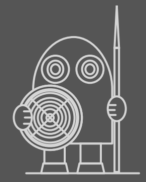
You can use the same approach with the other booklet. Start with the sheet you've got, with its four numbered pages, and work out what sheets (and what page-numbers) must have been above and below it :



As you can see here, there were 20 pages altogether in the original booklet – and the booklet was made from five single A4 sheets.

** Once again, the page-numbers on any sheet facing you always add up to the same thing – this time the total is 21 on every sheet. There's an obvious link between this total (21) and the number of pages in the booklet (20). Perhaps this would be a useful fact to know if you had to deal with a page from a much larger booklet . . .*



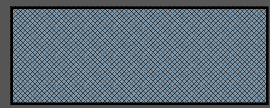


ans 47 action fractions

One way of getting to the answer is to use diagrams :

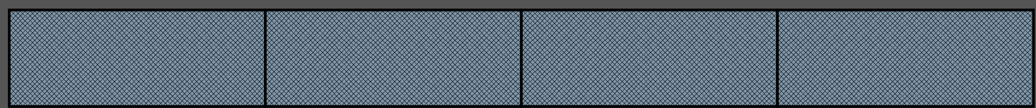
to begin with, let's use four boxes to stand for number 4, like this :

. . . and here's the action fraction a :



next, this shows 4 (four boxes) minus the action fraction a :

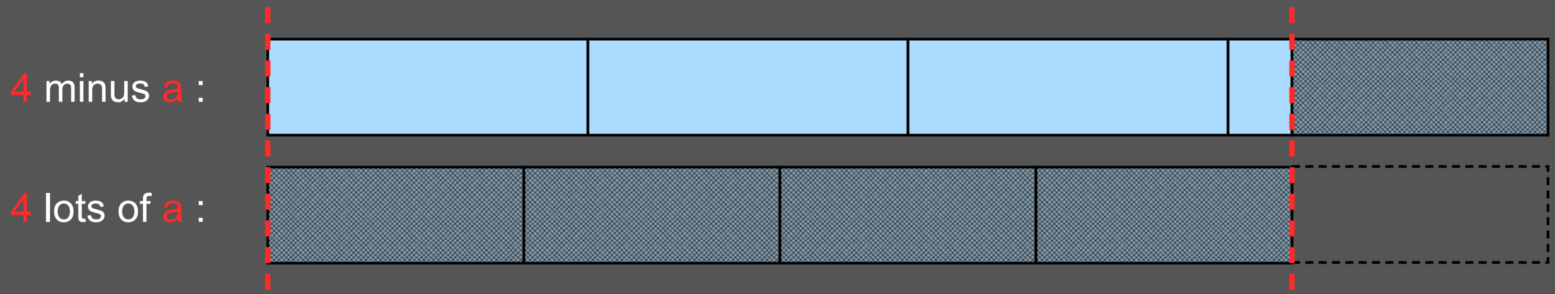
but we're told that this is the same as 4 lots of a :



PTO

action fractions

and we can show this fact by putting the two last diagrams together, as follows :



* Now for the clever bit : The last diagram shows that 4 lots of a is exactly the same as 4 minus a.

We can also see that the diagram shows that 5 lots of a are exactly the same as 4,

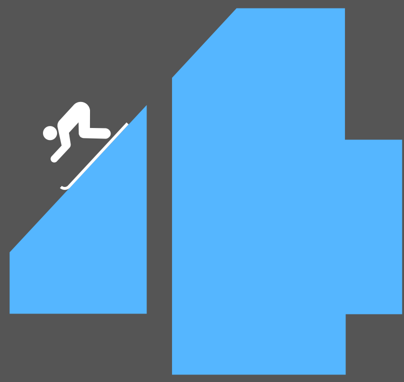
which of course means that a (the action fraction) must be 4 ÷ 5, that's to say $4/5$.

So, the action fraction for 4 is exactly $\underline{4/5}$

CHECK : *4 – 4/5 = 20/5 – 4/5 = 16/5 . . . and 4 x 4/5 = 16/5*

PTO

action fractions



Another way of going about the problem is this :

to begin with, let's call our action fraction a

we're told that 4 minus a is exactly the same as 4 lots of a

which must mean that 4 equals 5 lots of a

and that's the same as saying that a = 4/5

so the action fraction we're after is 4/5

* *if you know some algebra, you could show this way of getting the answer like this :*

$$4 - a = 4a$$

$$4 = a + 4a$$

$$4 = 5a$$

$$\underline{a = 4/5}$$

PTO ►►

action fractions

If you'd like to do a little more work on this topic, here are one or two more ideas you could investigate :

4/5

4/5 is the action fraction for 4 (because 4 minus 4/5 is the same as 4 times 4/5 : they both come to $3\frac{1}{5}$) but

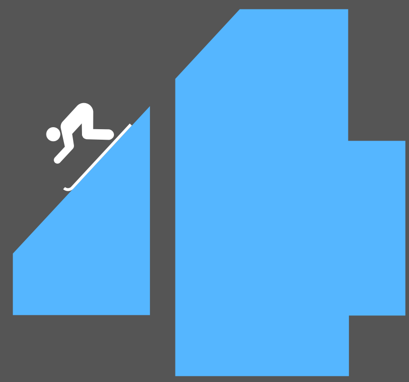
- What would be the action fraction for 2? or for 3? or for 5? or 6? There's a pattern here and it's not hard to find. How would you describe this pattern in words?

And if you're quite comfortable with fractions, you might like to try this (slightly harder) problem :

- Which fraction gives you the same result whether you subtract it from 1/4 or multiply it by 1/4 ? And what about other fractions like 1/2, 1/3 or 1/5? What's the action fraction for them? And what's the pattern?

PTO ⏩

ans 47 action fractions



Well, here are our results – and as you can see, the patterns in each one are easy to describe. Of course, there are plenty of other interesting (and sometimes surprising) things to discover about fractions . . .

number	action fraction
2	2/3
3	3/4
4	4/5
5	5/6
. . .	. . .

fraction	action fraction
1/2	1/3
1/3	1/4
1/4	1/5
1/5	1/6
. . .	. . .

ans 48 trip saver

Perhaps one way to try to solve this is by trial and improvement . . .

First of all, something useful to note : Liss is saving in multiples of 2 and Khori is saving in multiples of 3. This means that whatever the amount is when they're at the same total, it will have to be a multiple of 2 and of 3 : in other words, a multiple of 6 !

So now let's start looking at some multiples of 6 and note how long each of them would take to reach this amount, remembering that Liss began saving in Week 1 and Khori began saving in Week 9 (that's 8 weeks later) :

£30

khori : 10 weeks at £3 per week

liss : 15 weeks at £2 per week

10 weeks and 15 weeks differ by only 5 weeks

£36

khori : 12 weeks at £3 per week

liss : 18 weeks at £2 per week

12 weeks and 18 weeks differ by only 6 weeks

PTO ▶▶

ans 48 trip saver

£42

khori : 14 weeks at £3 per week

liss : 21 weeks at £2 per week

14 weeks and 21 weeks differ by only 7 weeks

£48

khori : 16 weeks at £3 per week

liss : 24 weeks at £2 per week

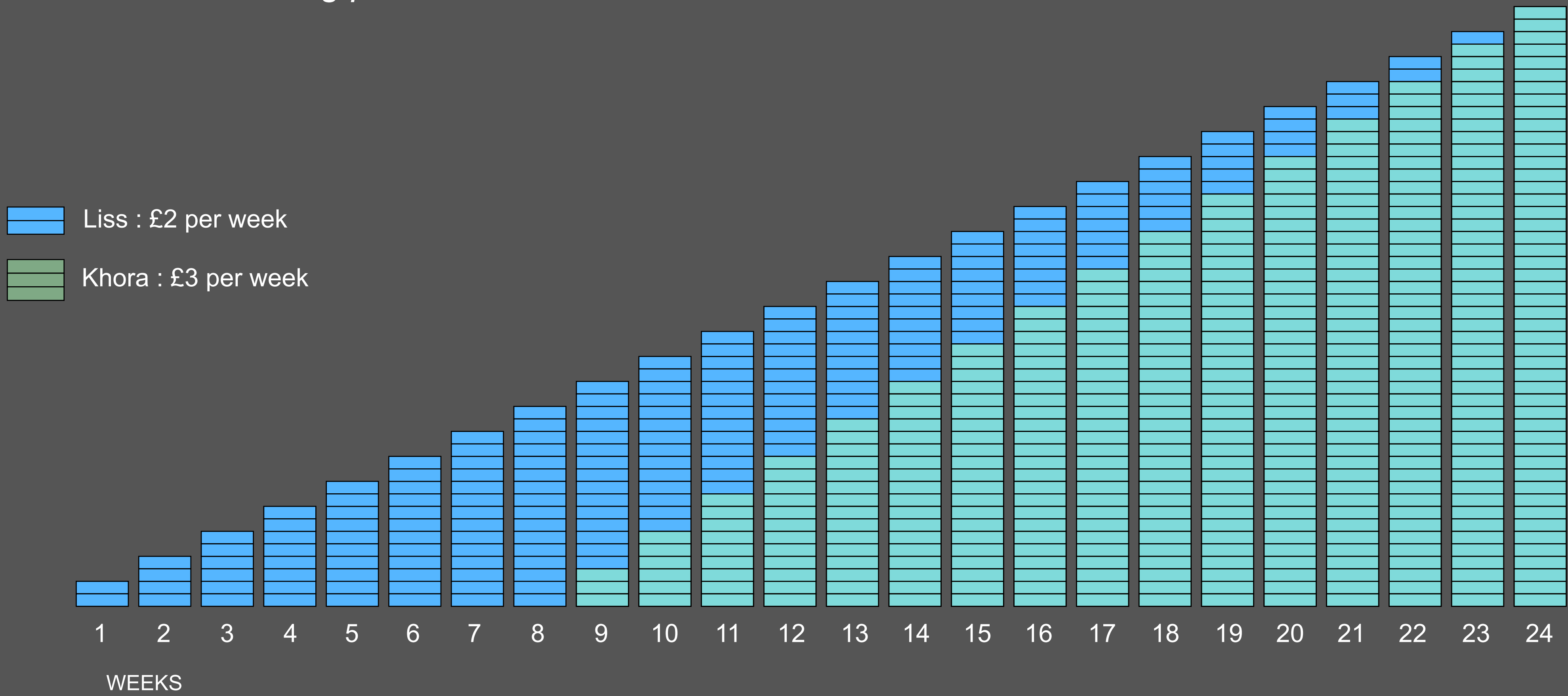
16 weeks and 24 weeks differ by 8 weeks — *and these are exactly the 8 weeks when khori hadn't started saving !*

So that's it! We've found that £48 is the amount at which the brother's and the sister's savings match – and we've found that this happens in week 24 of the year (that's Khori's 16th week of saving and Liss's 24th week of saving).

PTO ▶▶

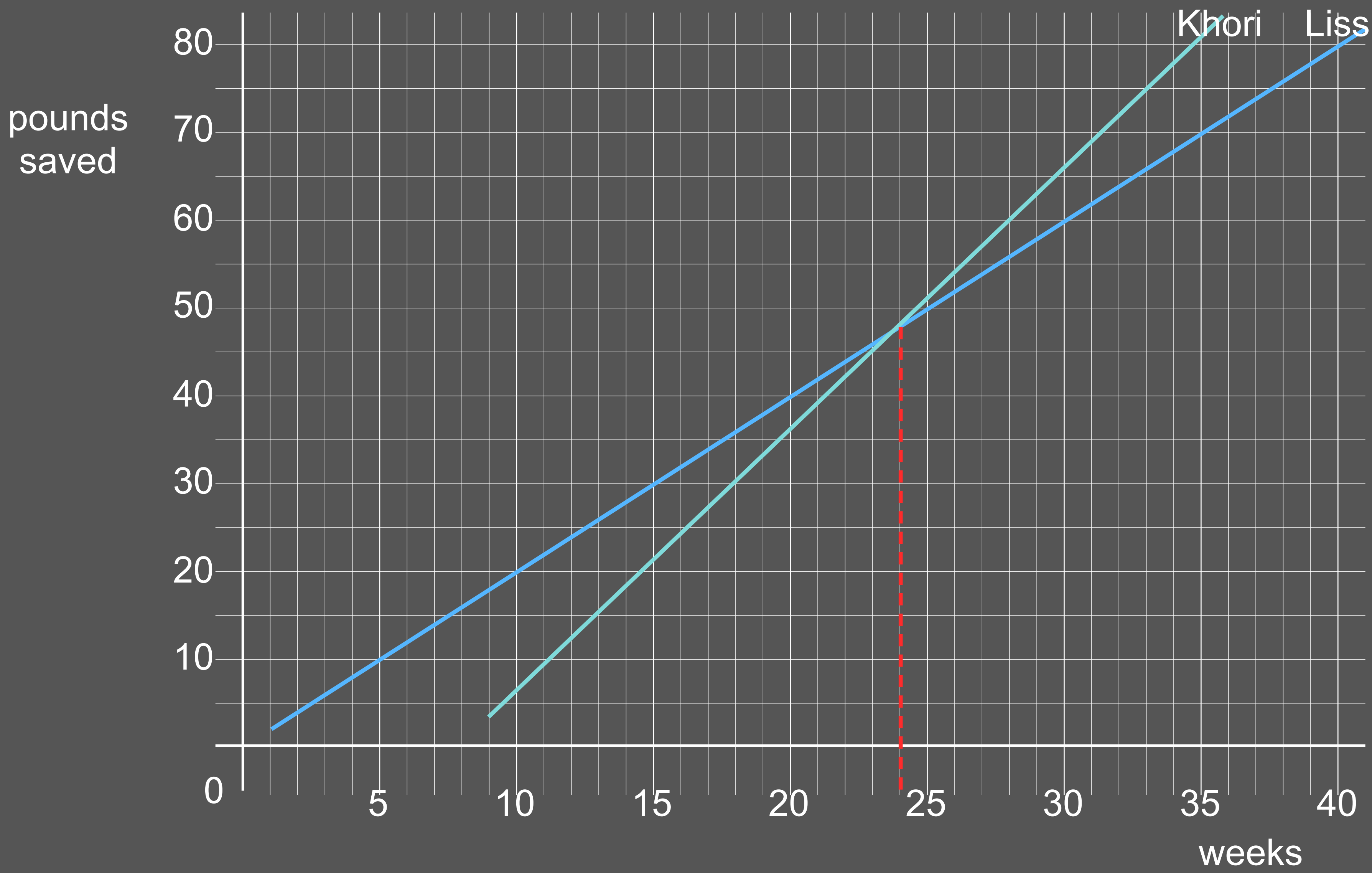
trip saver

Here and on the next page are two ways of showing the two saving patterns :



PTO

ans 48 trip saver

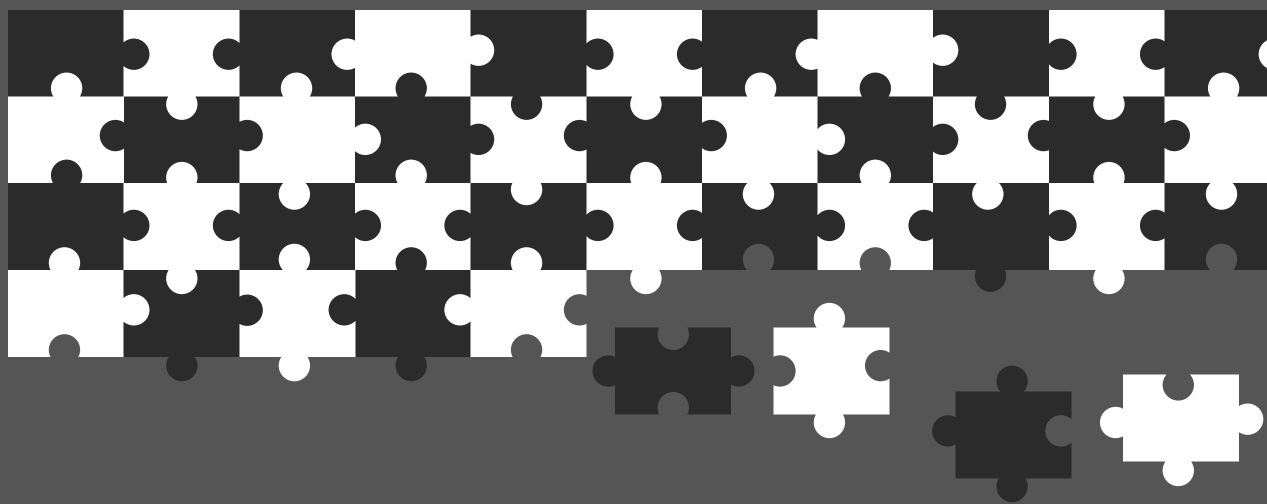


ANS 49 **black & white jigsaw . . .**

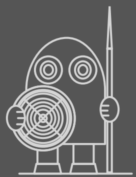
If you think about it, for a jigsaw to have one 'central piece', both its longer side and its shorter side must have an odd number of pieces. Syed's puzzle is 54 x 27, so as you can see, its longer side has an even number of pieces. This means that :

Syed's puzzle can't have one 'central piece'.

Now for the second question. Imagine putting this black-and-white jigsaw together. You would just lay down pieces along the top row, alternating black, white, black, white . . . and so on. Then you'd lay down pieces along the second row, alternating white, black, white, black . . . and so on. And you'd keep doing this all the way down to the bottom row, leaving the whole puzzle as a complete chequer-board of black and white :



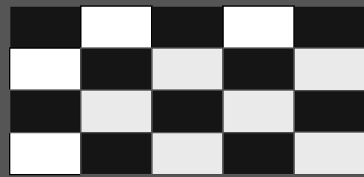
PTO



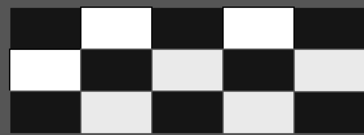
ans 49 black & white jigsaw . . .

And in fact, you'd find exactly the same number of black and white pieces in Syed's puzzle! But to see why this has to be true needs a bit of careful thinking. To begin with, we shall need to think about one or two different kinds of puzzle :

First of all, let's think of a puzzle with an even number of pieces in at least one direction. A simple 5 x 4 puzzle is one example. You can see straight away that the first **pair** of rows has 5 black and 5 white pieces – and so does the second **pair** of rows. Clearly, as long as we can split the puzzle into **pairs** of rows (or columns) like this, we'll always have the same total number of black and white pieces.



Next, we'll think of a puzzle with an odd number of pieces in each direction. A simple 5 x 3 puzzle is one example. As you can see, two of the rows have 3 black pieces and one row has 2 black pieces, making 8 black pieces in all. But only one row has 3 white pieces and two rows have just 2 white pieces, making 7 white pieces in all. It's clear that if we have an odd number of pieces in each direction, then there won't be the same number of black and white pieces.



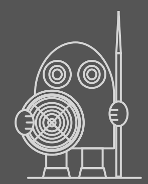
The black-and-white version of Syed's puzzle does have an even number of pieces in one direction – so his puzzle would have exactly the same number of black and white pieces! And this time it's the columns you can group in pairs.

ans 50 match days count

Look at the table below. Each column shows you one way of arranging matches for a league type of competition. As you can see, with 3 teams in a league you can organise things over 3 match-days (with 2 teams playing and 1 team resting on each day); and with 4 teams you can still fit the matches into 3 match-days (this time with all 4 teams in action on each match-day); with 5 teams you need 5 match-days.

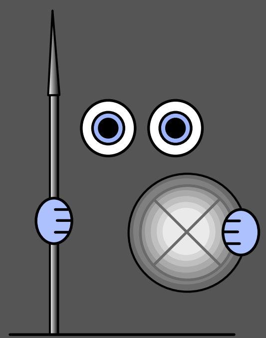
3 teams	*resting*	*4 teams*	*resting*	*5 teams*	*resting*
A B	*C*	*A B / C D*	*–*	*B C / D E*	*A*
B C	*A*	*A C / B D*	*–*	*A D / C E*	*B*
C A	*B*	*A D / B C*	*–*	*A E / B D*	*C*
				A C / B E	*D*
				A B / C D	*E*
= 3 match-days		*= 3 match-days*		*= 5 match-days*	

There are different ways of arranging the matches, but for 5 teams you will definitely need only 5 match-days.



no problems! book 2

50 maths problems with answers



four winds